The NLO Calculations of Heavy Quarkonium Production at B Factories

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Abstract.
We calculated the next to leading order quantum chromodynamics corrections of J/ψ production at B factories. The next to leading order corrections are very important for heavy quarkonium production at B factories. The next to leading order cross sections can be compared with the B factories data. Some detail of the next to leading order calculations within the non-relativistic quantum chromodynamics factorization framework are given here.

1. Introduction

It is widely believed that the production and annihilation of the heavy quarkonium can be described by non-relativistic quantum chromodynamics (NRQCD)[1]. Defferring from the case in the color-singlet model, in a color-octet production process, the intermediate pair of quark and antiquark can be created at short distances with different colors and then forms the non color quarkonium at long distances by emitting or absorbing soft gluons. The process at short distances called short-distance coefficients which can be calculated perturbatively, and the process at long distances dependents on non-perturbative NRQCD matrix elements which are universal, process-independence and must be determined by experimental extraction, potential model or lattice calculations.

The NRQCD factorization scheme seems to acquire some significant successes in describing heavy quarkonium decay and production. But recently, several next-to-leading order (NLO) QCD corrections for the inclusive and exclusive heavy quarkonium production in the color-singlet piece are found to be large and significantly, which relieve the conflicts between the color-singlet model predictions and experiments. It may imply, though inconclusively, that the color-octet contributions in the production processes are not as big as previously expected, and the color-octet mechanism should be studied more carefully. Lots of work have been done to investigate the color-octet mechanism in NRQCD for heavy quarkonium production[2, 3, 4, 5].

The charmonium production in e^+e^- annihilation at B factories has provided an important test ground for NRQCD and color-octet mechanism. The production of double charmonium in e^+e^- annihilation at B factories [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19] is one of the challenging problems in heavy quarkonium physics and NRQCD[1]. The large discrepancies of $J/\psi+\eta_c$ production in e^+e^- annihilation at B factories between LO theoretical predictions[9, 10, 11, 20, 21] and experimental results [7, 13] were challenging issues once but now are largely resolved by higher order corrections: NLO QCD[22, 23, 24, 25, 26, 27, 28, 29] and relativistic [19, 30, 18, 31, 32, 33, 34] corrections, and the results show that the color-singlet NLO corrections (both in α_s and v)[35, 36] may increase the cross section of double charmonium production e.g. $e^+e^- \to J/\psi\eta_c$ by an order of magnitude. However, the cross sections of other processes, i.e., $e^+e^- \to J/\psi(\psi(2S))\chi_{c0}$ measured by Belle[6] $\sigma[J/\psi+\chi_{c0}] \times B^{\chi_{c0}}[>2] = (16 \pm 5 \pm 4)$ fb are also larger than LO NRQCD predictions by about an order of magnitude or at least a factor of 5. Here $B^{\chi_{c0}}[>2]$ is the branching fraction for the χ_{c0} decay into more than 2 charged tracks. Theoretically, two studies in NRQCD by Braaten and Lee[9] and by Liu, He, and Chao[10] showed that, at LO in the QCD coupling constant α_s and the charm quark relative velocity v, the cross-section of $e^+e^- \to J/\psi(\psi(2S))\chi_{c0}$ at $\sqrt{s} = 10.6$GeV is only about $2.4 \sim 6.7(1.0 \sim 4.4)$fb (depending on the used parameters, e.g., the long-distance matrix elements, m_c and α_s). The QCD radiative correction can also greatly enhance $\sigma[J/\psi(\psi(2S))\chi_{c0}]$[24, 37, 38, 39, 37, 40].

On the inclusive J/ψ production, the total prompt J/ψ cross sections in e^+e^- annihilation were measured to be $\sigma_{tot} = 2.52 \pm 0.21 \pm 0.21$ pb by BaBar [41], whereas Belle gave a much smaller value $\sigma_{tot} = 1.47 \pm 0.10 \pm 0.13$ pb [42]. Obviously, the large discrepancy between the two measurements should be further clarified. Belle reported new measurements[43]

$$\sigma(e^+e^- \to J/\psi + X) = (1.17 \pm 0.02 \pm 0.07)pb, \tag{1}$$

$$\sigma(e^+e^- \to J/\psi + c\bar{c}) = (0.74 \pm 0.08^{+0.09}_{-0.08})pb, \tag{2}$$

$$\sigma(e^+e^- \to J/\psi + X_{non\ c\bar{c}}) = (0.43 \pm 0.09 \pm 0.09)pb. \tag{3}$$

The J/ψ inclusive production cross section given in Eq.(1) is significantly smaller than that given by BaBar[41] and Belle[42] previously. The double charm production cross section given in Eq.(2) also becomes smaller accordingly. However, the ratio of J/ψ production rate through double $c\bar{c}$ to that of J/ψ inclusive production measured by Belle[7, 43]

$$R_{c\bar{c}} = \frac{\sigma[e^+e^- \to J/\psi + c\bar{c}]}{\sigma[e^+e^- \to J/\psi + X]} \quad = \quad 0.59^{+0.15}_{-0.13} \pm 0.12, \tag{4}$$

are much larger than LO NRQCD predictions. If only including the color-singlet contribution at LO in α_s, the ratio is about $0.2 \sim 0.4$[44].

The cross section of $J/\psi+X_{non\ c\bar{c}}$ includes the color-singlet contribution of $e^+e^- \to J/\psi+gg$ and the color-octet one of $e^+e^- \to c\bar{c}({}^3P_0^{(8)}$ or ${}^3P_0^{(8)})g$ at leading order of α_s. The color-singlet piece has been investigated by including the NLO $O(\alpha_s)$ correction[29, 28] and $O(v^2)$ relativistic correction[32, 33], of which each contributes an enhancement factor of $1.2-1.3$ to the cross section of $e^+e^- \to J/\psi + gg$. As a result, the color-singlet contribution has saturated the observed value given in Eq.(3), leaving little room for the color-octet contribution.

In the color octet J/ψ inclusive production $e^+e^- \to J/\psi + X$ at B factories, there are two processes. One is $e^+e^- \to q\bar{q} + J/\psi + X(q = u, d, s)$, which was found to be negligible at $\sqrt{s} = 10.6$ GeV and can only be important at much higher energies than $\sqrt{s} = 10.6$ GeV[44]. The other color-octet process $e^+e^- \to c\bar{c}({}^1S_0^{(8)}$ or ${}^3P_J^{(8)})g$ was studied by Braaten and Chen[45]. Based on the LO NRQCD calculation, they predicted that the J/ψ production is dominated in the region near the upper endpoint in the J/ψ energy distribution, and the width of the peak near the endpoint is of the order of 150 MeV. But the measured J/ψ spectra in e^+e^- annihilation by BaBar [41] and Belle[42] do not exhibit any enhancement near the endpoint. And a large color-octet contribution to the J/ψ inclusive production would enhance the denominator and then further decrease this ratio. So, this became a very puzzling issue. Some theoretical studies have been suggested in resolving this problem. Fleming, Leibovich and Mehen use the Soft-Collinear Effective Theory (SCET) to resum the color-octet contribution[46]. Lin and Zhu use SCET to analyze the color-singlet contribution to $e^+e^- \to J/\psi gg$ [47]. Leibovich and Liu sum the leading and next-to-leading logarithms in the color-singlet contribution to the J/ψ production cross section[48]. As a new step, Zhang and Chao find the NLO QCD corrections to $e^+e^- \to J/\psi + c\bar{c}$[23] to be large, and increase the cross sections by a factor of about 2 (using the same matrix elements as LO), making the ratio R larger than the LO results.

In order to further clarify this problem, it is certainly useful to study the the higher order corrections of J/ψ production at B factories. We have calculated $e^+e^- \to J/\psi + \eta_c$[22], $e^+e^- \to J/\psi + \chi_{c0}$[24], $e^+e^- \to J/\psi + c\bar{c}$[23], $e^+e^- \to J/\psi + gg$[29], and $e^+e^- \to J/\psi + g$[49]. The paper is organized as follows. In Section II, we will give the frame of the calculation. In Section III , we will give the detail of the loop integrate. In section IV, we will give the numerical results and relations to the color-octet matrix elements. A summary will be given in Section V.

2. The frame of the calculation

In leading order in strong coupling constant α_s, for example, $J/\psi + \eta_c$ can be produced at order $\alpha^2\alpha_s^2$. We refer to Ref [10]. There are four feynman diagrams, two of which are shown in Fig. 1. The other two diagrams are gotten through inversing quark lines. Momenta for the involving particles are assigned as $e^-(k_1)e^+(k_2) \to J/\psi(2p_1) + \eta_c(2p_2)$. We now write down the scattering amplitude in the nonrelativistic approximation to describe the creation of two color-singlet $c\bar{c}$ pairs which subsequently hadronize to two charmonium states in the e^+e^- annihilation process in Fig. 1

$$
\begin{aligned}
\mathcal{A}(a+b \to Q\bar{Q}(^{2S_\psi+1}L_{J_\psi})(p_3) + Q\bar{Q}(^{2S+1}L_J)(p_4)) &= \sqrt{C_{L_\psi}}\sqrt{C_L}\sum_{L_{\psi z}S_{\psi z}}\sum_{s_1s_2}\sum_{jk}\sum_{L_zS_z}\sum_{s_3s_4}\sum_{il} \\
&\times \langle s_1; s_2 \mid S_\psi S_{\psi z}\rangle\langle L_\psi L_{\psi z}; S_\psi S_{\psi z} \mid J_\psi J_{\psi z}\rangle\langle 3j; \bar{3}k \mid 1\rangle \\
&\times \langle s_3; s_4 \mid SS_z\rangle\langle LL_z; SS_z \mid JJ_z\rangle\langle 3l; \bar{3}i \mid 1\rangle \\
&\times \begin{cases} \mathcal{A}(a+b \to Q_j(\frac{p_3}{2}) + \bar{Q}_k(\frac{p_3}{2}) + Q_l(\frac{p_4}{2}) + \bar{Q}_i(\frac{p_4}{2})) & (L=S), \\ \epsilon^*_\alpha(L_Z)\mathcal{A}^\alpha(a+b \to Q_j(\frac{p_3}{2}) + \bar{Q}_k(\frac{p_3}{2}) + Q_l(\frac{p_4}{2}) + \bar{Q}_i(\frac{p_4}{2})) & (L=P), \end{cases}
\end{aligned} \tag{5}
$$

where $\langle 3j; \bar{3}k \mid 1\rangle = \delta_{jk}/\sqrt{N_c}$, $\langle 3l; \bar{3}i \mid 1\rangle = \delta_{li}/\sqrt{N_c}$, $\langle s_1; s_2 \mid S_\psi S_{\psi z}\rangle$, $\langle s_3; s_4 \mid SS_z\rangle$, $\langle L_\psi L_{\psi z}; S_\psi S_{\psi z} \mid J_\psi J_{\psi z}\rangle$ and $\langle LL_z; SS_z \mid JJ_z\rangle$ are respectively the color-SU(3), spin-SU(2), and angular momentum Clebsch-Gordan coefficients for $Q\bar{Q}$ pairs projecting out appropriate bound states. $\mathcal{A}(a+b \to Q_j(\frac{p_3}{2}) + \bar{Q}_k(\frac{p_3}{2}) + Q_l(\frac{p_4}{2}) + \bar{Q}_i(\frac{p_4}{2}))$ is the scattering amplitude for double $Q\bar{Q}$ production and $\mathcal{A}^\alpha$ is the derivative of the amplitude with respect to the relative momentum between the quark and anti-quark in the bound state. The coefficients C_{L_ψ} and C_L can be related to the radial wave function of the bound states or its derivative with respect to the relative spacing as

$$
\begin{aligned}
C_s &= \frac{1}{4\pi}|R_s(0)|^2, \\
C_p &= \frac{3}{4\pi}|R'_p(0)|^2
\end{aligned} \tag{6}
$$

We introduce the spin projection operators $P_{SS_z}(p,q)$ as[10]

$$
P_{SS_z}(2p,q) \equiv \sum_{s_1s_2}\langle s_1; s_2|SS_z\rangle c(p-q;s_1)\bar{c}(p+q;s_2). \tag{7}
$$

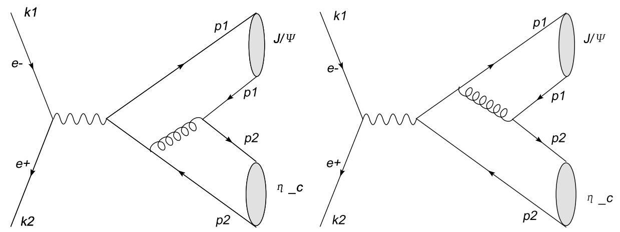

Figure 1. Two of the four born diagrams for $e^-(k_1)e^+(k_2) \to J/\psi(2p_1)\eta_c(2p_2)(\chi_{c0}(2p_2))$.

Expanding $P_{SS_z}(P, q)$ in terms of the relative momentum q, we get the projection operators and their derivatives, which will be used in our calculation, as follows

$$
\begin{aligned}
P_{00}(2p, q) &= \sum_{s\bar{s}} v(s)\bar{u}(\bar{s})\langle\frac{1}{2}, s; \frac{1}{2}, \bar{s}|0, 0\rangle \\
&= \frac{1}{2\sqrt{2}(E_q + m)}(-\not{p}_{\bar{c}} + m_c)\gamma_5 \frac{\not{P} + 2E_q}{2E_q}(\not{p}_c + m_c).
\end{aligned}
\tag{8}
$$

For spin-triplet case, the expression is defined as

$$
\begin{aligned}
P_{1S_z}(2p, q) &= \sum_{s\bar{s}} v(s)\bar{u}(\bar{s})\langle\frac{1}{2}, s; \frac{1}{2}, \bar{s}|1, S_z\rangle \\
&= \frac{1}{2\sqrt{2}(E_q + m)}(-\not{p}_{\bar{c}} + m_c)\not{\epsilon} \frac{\not{P} + 2E_q}{2E_q}(\not{p}_c + m_c),
\end{aligned}
\tag{9}
$$

In next-to-leading order in strong coupling constant α_s,the cross section is

$$
\sigma = |\mathcal{M}_{Born} + \mathcal{M}_{NLO}|^2 = |\mathcal{M}_{Born}|^2 + 2\mathrm{Re}(\mathcal{M}_{Born}\mathcal{M}^*_{NLO}) + \mathcal{O}(\alpha^2\alpha_s^4)
\tag{10}
$$

Then we need to calculate $\mathcal{M}_{NLO}$. We use `FeynArts` [50, 51] to generate feynman diagrams, `FeynCalc` [52] for the tensor reduction, and `LoopTools` [53] for the numerical evaluation of the IR-safe integrals. One-loop diagrams contain $\gamma \to gg, g \to J/\psi, g \to \eta_c, gg \to J/\psi$ will vanish for color and charge conjugation. Then the self-energy, triangle diagrams are all correspond to propagators and vertexes of born diagrams. And it remains twenty-four box and pentagon diagrams. Twelve diagrams of them are shown in Fig. 2. The upper $c\bar{c}$ hadronize to J/ψ which momentum is $2p_1$,and the lower $c\bar{c}$ hadronize to η_c which momentum is $2p_2$. The other twelve diagrams are gotten through inversing quark lines. Specially, the associated diagram of Pentagon N12 is only inversing the lower quark line which hardonize to η_c, and inverse all quark lines get itself.

The self-energy and triangle diagrams are in general UV divergent; the triangle, box, and pentagon diagrams are in general IR divergent. Box N5 and N8 and Pentagon N10 ,which have a virtual gluon line connecting the $c\bar{c}$ in a meson, also contain Coulomb singularities, which are cancelled after taking into account the corresponding corrections to the operators $\left\langle \mathcal{O}^{J/\psi}\left[{}^3S_1^{(1)}\right]\right\rangle$ and $\left\langle \mathcal{O}^{\eta_c}\left[{}^1S_0^{(1)}\right]\right\rangle$. In the practical calculation, the IR and UV singularities are regularized with $D = 4 - 2\epsilon$ space-time dimension, and the Coulomb singularities are first regularized by a small relative velocity v between the $c\bar{c}$ [54]. $v = |\overrightarrow{p_{1c}} - \overrightarrow{p_{1\bar{c}}}|/m$, defined in meson c.m. frame. For the

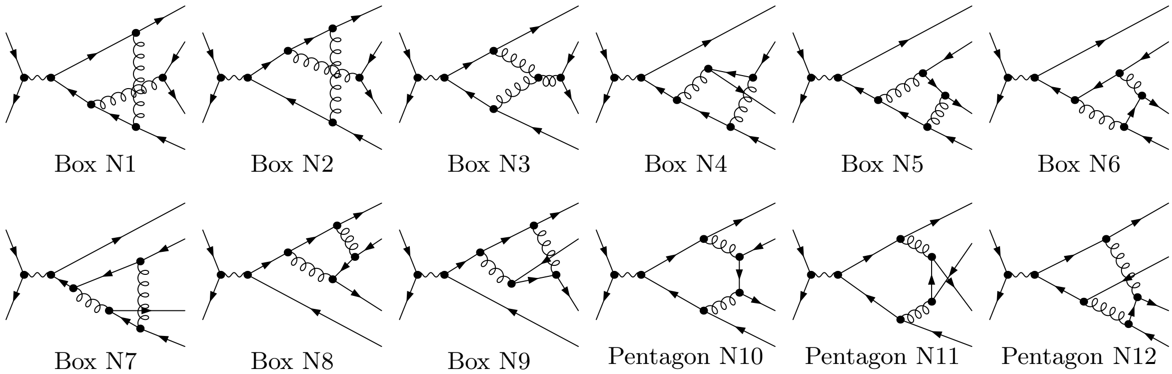

Figure 2. Twelve of the twenty-four box and pentagon diagrams.

Coulomb-singularity part of the virtual cross section, we find

$$\sigma \;=\; |R_S(0)|^4\hat{\sigma}^{(0)}\left(1+\frac{2\pi\alpha_s C_F}{v}+\frac{\alpha_s\hat{C}}{\pi}+\mathcal{O}(\alpha_s^2)\right), \tag{11}$$

where the color factor is given by $C_F = 4/3$ in $SU(3)_c$. Leading order of operators $\left\langle \mathcal{O}^{J/\psi}\left[{}^3S_1^{(1)}\right]\right\rangle$ and $\left\langle \mathcal{O}^{\eta_c}\left[{}^1S_0^{(1)}\right]\right\rangle$ are associated with $R_S(0)$, and next-to-leading order are $\pi\alpha_s C_F/v$ [1]. And two operators give a factor of 2 at $\mathcal{O}(\alpha_s)$, just the Coulomb-singularity term in Eq. (11). Then the corresponding contribution of Coulomb-singularity has to be factored out and mapped into the wave functions of J/ψ and η_c:

$$\begin{aligned}\sigma \;&=\; |R_S(0)|^4\left(1+\frac{2\alpha_s}{\pi}\,\frac{C_F\pi^2}{v}\right)\hat{\sigma}^{(0)}\left[1+\frac{\alpha_s}{\pi}\hat{C}+\mathcal{O}(\alpha_s^2)\right]\\ &\Rightarrow\; |R_S(0)|^4\;\hat{\sigma}^{(0)}\left[1+\frac{\alpha_s}{\pi}\hat{C}+\mathcal{O}(\alpha_s^2)\right].\end{aligned} \tag{12}$$

Selecting feynman gauge, then we get the amplitude of born diagrams

$$i\mathcal{M}_{Born} = \frac{4096\pi e_c\alpha\alpha_s m|R_s(0)|^2}{3s^3}\;\epsilon_{\alpha\beta\nu\rho}p_1^{\alpha}p_2^{\beta}\varepsilon^{*\nu}\bar{v}_e(k_2)\gamma^{\rho}u_e(k_1), \tag{13}$$

where $s = (k_1 + k_2)^2$, $e_c = \frac{2}{3}$ is the fractional electric charge of the charm quark. The vector indices ρ is for the virtual photon. ε is the polarization vector of J/ψ. p_1 and p_2 are half momenta of J/ψ and η_c respectively. The coefficients $R_S(0)$ is the radial wave function at origin of the bound states.

3. Detail of the calculation

There are only two independent momentum p_1, p_2 in the process of $\gamma^* \to J/\psi + \eta_c(\chi_{c0})$, but the five points loop integral will be

$$\int \mathrm{d}^D q\frac{1}{N_0^2N_1N_2N_3N_4}, \tag{14}$$

For the number of independent momentum is less than 4, the $GramDeterminant = 0$[55]. And the traditional Passarino-Veltman reduction does not work here. The first steps is separation the IR divergence[56]. Then we can introduce the mass of gluon and solve the equation

$$\sum_{i=0}^{4} a_i N_i / C = 1$$
$$\int \frac{\mathrm{d}^D q}{N_0 N_1 N_2 N_3 N_4} = \sum_{i=0}^{4} \frac{a_i}{C} \int \frac{N_i \mathrm{d}^D q}{N_0 N_1 N_2 N_3 N_4}. \tag{15}$$

where $C \neq 0$ is independent on integral momentum q and a_i is constant. They become four points integral and can be reduced again. Then we can get a set of three points integral and calculate directly.

If we do not introduce the mass of gluon, $C = 0$ when the gluon connect with the both legs of J/ψ. Then we can reduce the integral to a set of basic three, four, and five points function. The five points integral is the integral which appears in N10 of Fig.2

$$\begin{aligned}
& E_0^{fin}[p_1, 2p_1, -p_2, -2p_2, m, 0, m, 0, m] \\
&= E_0 - \frac{2}{s} D_0[-p_1, -p_1 - p_2, p_1, 0, m, 0, m] - \frac{2}{s} D_0[p_1 \leftrightarrow p_2] \\
&= \int \frac{\mathrm{d}^D q/(2\pi)^D \left(s/2 - 2(q^2 - m^2) - 4q\cdot p_1 + 4q \cdot p_2 - 8m^2\right) 2/s}{(q^2 - m^2)(q + p_1)^2((q + 2p_1)^2 - m^2)(q - p_2)^2((q - 2p_2)^2 - m^2)} \\
&= \frac{-4}{s} D_0[p_1 + p_2, p_1 + 2p_2, -p_1, 0, 0, m, m] + \int \frac{\mathrm{d}^D q}{(2\pi)^D} \\
& \frac{2/s(s/2 - 4q\cdot p_1 + 4q\cdot p_2 - 8m^2)}{(q^2 - m^2)(q + p_1)^2((q + 2p_1)^2 - m^2)(q - p_2)^2((q - 2p_2)^2 - m^2)} \\
&= \text{First Term} + \int \frac{\mathrm{d}^D q}{(2\pi)^D} \int_0^1 \\
& \frac{\Pi_{i=1}^5 \mathrm{d}x_i \delta(\sum_{j=1}^5 x_j - 1) 4! (1 - 16m^2/s)(1 - X - Y)}{[(q + Xp_1 - Yp_2)^2 - m^2(1 - X - Y)^2 + XYs/4]^5}
\end{aligned}$$

where $X = x_1 + 2x_2, Y = x_3 + 2x_4$. The `First Term` is IR- and Coulomb-finite. It can be calculated in $D = 4$ space-time dimension and $v = 0$, it is

$$\frac{2\sqrt{4m^2 - s}\,\tan^{-1}\frac{\sqrt{s}}{\sqrt{4m^2-s}} - \sqrt{s}\,\ln\frac{-s}{m^2}}{-i\pi^2 m^2 s^{5/2}} \tag{16}$$

The second term is IR- and Coulomb-finite too. Choose $\{x_1, x_2, x_3, x_4, x_5\} = \{1–a, ab(1–c), a(1–b), abcd, abc(1–d)\}$ and integral a, b, d, c step by step in `Mathematica`,

$$\frac{2(4m^2 - s)^{3/2}\tan^{-1}\frac{\sqrt{s}}{\sqrt{4m^2-s}} + \sqrt{s}\left(i\pi(3m^2 - s) + (s - 4m^2)\ln\frac{-s}{m^2}\right)}{8im^4\pi^2(4m^2 - s)s^{5/2}(16m^2 - s)^{-1}} \tag{17}$$

and $\ln(-s/m^2) = \ln(-(s + i0)/m^2) = \ln(s/m^2) - i\pi$. $D_0[-p_1, -p_1 - p_2, p_1, 0, m, 0, m]$ term in Eq. (16) is,

$$D_0[-p_1, -p_1 - p_2, p_1, 0, m, 0, m] = \frac{4}{s} C_0[-p_1, p_1, 0, m, m] + \frac{i}{(4\pi)^2} \frac{2i\pi - 2\ln 4}{m^2 s}. \tag{18}$$

This term will appear in Box N5, N8.

$$C_0[p_{1c}, -p_{1\bar{c}}, 0, m, m] = \frac{-i}{2m^2(4\pi)^2} \left(\frac{4\pi\mu^2}{m^2}\right)^{\epsilon} \Gamma(1 + \epsilon) \left[\frac{1}{\epsilon} + \frac{\pi^2}{v} - 2\right] \tag{19}$$

where $v = |\overrightarrow{p_{1c}} - \overrightarrow{p_{1\bar{c}}}|/m$, defined in meson c.m. frame. For the $J/\psi + \chi_{c0}$, the corresponded integral is

$$\int \frac{\mathrm{d}^D q}{N_0^2 N_1 N_2 N_3 N_4}. \tag{20}$$

It can be integrald analysis too[24].

In another way, we can introduce the equation

$$\sum_{i=1}^{4} a_i \frac{N_i}{N_0} \quad = \quad 1, \tag{21}$$

where a_i is constant. Then we get

$$\int \frac{\mathrm{d}^D q}{N_0 N_1 N_2 N_3 N_4} = \sum_{i=1}^{4} a_i \int \frac{N_i \mathrm{d}^D q}{N_0^2 N_1 N_2 N_3 N_4}. \tag{22}$$

They become four points integral but N_0^2. We can repeat the process, and get a set of three points integral such as $\int \frac{\mathrm{d}^D q}{N_0^3 N_1^2 N_2}$, $\int \frac{\mathrm{d}^D q}{N_0^2 N_1^2 N_2^2}$, and so on. This integral can be calculated by Integration-By-Parts (IBP) method [57, 58, 59, 60].

4. Numerical result

In the numerical result, we select $m_{J/\psi} = m_{\eta_c} = 2m$ $\Lambda_{\overline{\mathrm{MS}}}^{(4)} = 338$MeV, $m = 1.5$GeV, $\mu = 2m$, the NLO cross section is

$$\sigma(e^+ + e^- \to J/\psi + \eta_c) = 15.7\mathrm{fb}, \tag{23}$$

which is larger than LO cross section 8.0 fb by a factor of 1.96. It is can be compared with the B factories data 17 $\sim$ 25 fb. And

$$\sigma(e^+ + e^- \to J/\psi + \chi_{c0}) \quad = \quad 10.1\mathrm{fb}, \tag{24}$$

which are a factor of 1.6 larger than the LO cross sections 6.35 fb.

If we select $m_c = 1.4$ GeV and μ=$2m$, the cross section at for $J/\psi + c\bar{c}$ is

$$\sigma(e^+ + e^- \to J/\psi + c\bar{c} + X) = 0.47 \text{ pb}. \tag{25}$$

It is about a factor of 1.7 larger than leading order cross section 0.27 pb. Including the contributions of the higher charmonium transition of other contributions, then the prompt cross section is

$$\sigma_{prompt}(e^+ + e^- \to J/\psi + c\bar{c} + X) = 0.71 \text{ pb}. \tag{26}$$

It is about 96% of the new Belle date 0.74 pb. The cross sections of prompt (feeddown included) $J/\psi gg$ and $J/\psi c\bar{c}$ production in e^+e^- annihilation at B factories are shown in Tab.1.

For the numerical calculations of the color octet cross sections, we select $m_c = 1.55$ GeV and μ=$2m$. The cross section at LO in α_s is

$$\sigma(e^+ + e^- \to J/\psi + X) \quad = \quad \Big[11 \frac{\left\langle 0 \left| \mathcal{O}^{J/\psi}[^1S_0^{(8)}] \right| 0 \right\rangle}{\mathrm{GeV}^3} + 18 \frac{\left\langle 0 \left| \mathcal{O}^{J/\psi}[^3P_0^{(8)}] \right| 0 \right\rangle}{\mathrm{GeV}^5}\Big] \mathrm{pb}, \tag{27}$$

while the cross section at NLO in α_s becomes

$$\sigma(e^+ + e^- \to J/\psi + X) \quad = \quad \Big[21 \frac{\left\langle 0 \left| \mathcal{O}^{J/\psi}[^1S_0^{(8)}] \right| 0 \right\rangle}{\mathrm{GeV}^3} + 35 \frac{\left\langle 0 \left| \mathcal{O}^{J/\psi}[^3P_0^{(8)}] \right| 0 \right\rangle}{\mathrm{GeV}^5}\Big] \mathrm{pb}. \tag{28}$$

Table 1. Cross sections of prompt (feeddown included) $J/\psi gg$ and $J/\psi c\bar{c}$ production in e^+e^- annihilation at B factories in units of pb.

	Belle Data	$\mu = 2.8$ GeV LO	$\mu = 2.8$ GeV NLO	$\mu = 5.3$ GeV LO	$\mu = 5.3$ GeV NLO
$\sigma(gg)$	0.43	0.57	0.67	0.36	0.53
$\sigma(c\bar{c})$	0.74	0.38	0.71	0.24	0.53
$R_{c\bar{c}}$	0.63	0.40	0.51	0.40	0.50

The NLO short-distance coefficients are larger than the LO coefficients by a factor of about 1.9. The color-octet matrix element

$$M_k = \langle 0|\mathcal{O}^{J/\psi}[^1S_0^{(8)}]|0\rangle + k\,\langle 0|\mathcal{O}^{J/\psi}[^3P_0^{(8)}]|0\rangle/m_c^2 \tag{29}$$

In fact, for the $e^+e^- \to J/\psi + X_{non\ c\bar{c}}$ production, the color-singlet process $e^+e^- \to J/\psi + gg$ has been found to make a dominant contribution to the cross section: $\sigma(e^+e^- \to J/\psi + gg) = 0.4 \sim 0.7$ pb at NLO in α_s and v^2 [29, 28, 32, 33], thus leaves little room to the color-octet contributions. This gives a very stringent constraint on the color-octet contribution, and may imply that the values of color-octet matrix elements are much smaller than expected earlier by using the naive velocity scaling rules or extracted from fitting experimental data with the leading-order calculations. Even if we disregard the dominant contribution of $e^+e^- \to J/\psi + gg$ to $e^+e^- \to [J/\psi + X_{non\ c\bar{c}}]$ by setting the color-singlet contribution $\sigma(e^+e^- \to J/\psi + gg)$ to be zero, and combining Eq.(3) with Eq.(28), we can get an upper bound of the color-octet matrix element:

$$M_{4.0}^{NLO} \quad < \quad (2.0 \pm 0.6) \times 10^{-2}\ \mathrm{GeV}^3. \tag{30}$$

5. Summary

We calculated the NLO QCD corrections of J/ψ production at B factories. The NLO corrections are very important for heavy quarkonium production at B factories. The NLO cross sections can be compared with the B factories data. Some detail of the NLO calculations within the NRQCD factorization framework are given here.

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