

# Computation of multi-leg amplitudes with NJET

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**Abstract.** In these proceedings we report our progress in the development of the publicly available C++ library NJET for accurate calculations of high-multiplicity one-loop amplitudes. As a phenomenological application we present the first complete next-to-leading order (NLO) calculation of five jet cross section at hadron colliders.

## 1. Introduction

NLO predictions of multi-jet production at hadron colliders have a long history. They are important processes for the LHC both as precision tests of QCD and direct probes of the strong coupling and also as background in many new physics searches. The LHC experiments have been able to measure jet rates for up to 6 hard jets which are now being used in new physics searches [1–3]. This presents a serious challenge for precise theoretical predictions since high multiplicity computations in QCD are notoriously difficult. Di-jet production has been known at NLO for more than 20 years [4] and has recently seen improvements via NLO plus parton shower (NLO+PS) description [5, 6] and steady progress towards NNLO QCD results [7]. The full three-jet computation was completed and implemented in a public code NLOJET++ 10 years ago [8]. Recently predictions for four-jet production have been presented by two independent groups [9, 10].

The advances in methods of evaluation of multi-leg virtual amplitudes [11–25] have inspired many efforts to automate NLO computations [26–31]. Processes with four final states, previously out of reach, can now be routinely used for phenomenological predictions [32–38]. We refer the reader to other contributions to these proceedings for further details on the current state-of-the-art [39–41].

Five partons in the final state still constitute a considerable challenge, though steady progress in that direction gives hope for the same level of automation in the near future. Recent state-of-the-art calculations with five QCD partons in the final state include the NLO QCD corrections to  $pp \rightarrow W + 5j$  [34] by the BLACKHAT collaboration and NLO QCD corrections to  $pp \rightarrow 5j$  [42].

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## 2. 5-Jet production at the LHC at 7 and 8 TeV

The different parts of the calculation, which contribute to NLO cross section can be schematically written as

$$\delta\sigma^{\text{NLO}} = \int_n (d\sigma_n^{\text{V}} + \int_1 d\sigma_{n+1}^{\text{S}}) + \int_n d\sigma_n^{\text{Fac}} + \int_{n+1} (d\sigma_{n+1}^{\text{R}} - d\sigma_{n+1}^{\text{S}}). \quad (1)$$

We used the Sherpa Monte-Carlo event generator [43] to handle phase-space integration and generation of tree-level amplitudes and Catani-Seymour dipole subtraction terms as implemented in Comix [44, 45].

The one-loop matrix elements for the virtual corrections  $d\sigma_n^{\text{V}}$  are evaluated with the publicly available NJET<sup>6</sup> package [31] interfaced to Sherpa via the Binouh Les Houches Accord [46, 47]. NJET is based on the NGLUON library [26] and uses an on-shell generalized unitarity framework [17–22] to compute multi-parton one-loop primitive amplitudes from tree-level building blocks [48]. The scalar loop integrals are obtained via the QCDLOOP/FF package [49, 50].

NJET implements full-colour expressions for up-to five outgoing QCD partons. The complexity of high-multiplicity virtual corrections motivates us to explore ways to speed up the computation. One of the optimizations implemented in NJET is the usage of de-symmetrized colour sums for multi-gluon final states, which allows us to get full colour result at a small fraction of computational cost by exploiting the Bose symmetry of the phase space [31, 51]. Another possibility is to separate leading and sub-leading contributions, which enables Monte-Carlo integrator to sample the dominant however simpler terms more often and get the same statistical error with fewer evaluations of the expensive sub-leading part.

In our leading terms we include all multi-quark processes in the large  $N_c$  limit and processes with two or more gluons in the final state using the de-symmetrized colour sums. In Figure 1 we compare leading and full virtual contributions to the hadrest jet transverse momentum in  $pp \rightarrow 5j$ . The correction from the sub-leading part is around 10% at low  $p_T$  and shows a tendency to grow with increasing hardness of the jet. Considering that  $d\sigma_n^{\text{V}}$  contribute  $\sim 50\%$  of the total NLO cross section for this process this translates to 5 – 10 percent effect depending on the kinematic region.

The calculation is done in QCD with five massless quark flavours including the bottom-quark in the initial state. We neglect contributions from top quark loops. We set the renormalization scale equal to the factorization scale ( $\mu_r = \mu_f = \mu$ ) and use a dynamical scale based on the total transverse momentum  $\widehat{H}_T$  of the final state partons:

$$\widehat{H}_T = \sum_{i=1}^{N_{\text{parton}}} p_{T,i}^{\text{parton}}. \quad (2)$$

For the definition of physical observables we use the anti-kt jet clustering algorithm as implemented in FASTJET [52, 53]. We apply asymmetric cuts on the jets ordered in transverse momenta,  $p_T$ , to match the ATLAS multi-jet measurements [1]:

$$p_T^{j_1} > 80 \text{ GeV} \quad p_T^{j_{\geq 2}} > 60 \text{ GeV} \quad R = 0.4 \quad (3)$$

The PDFs are obtained through the LHAPDF interface [54] with all central values using NNPDF2.1 [55] for LO ( $\alpha_s(M_Z) = 0.119$ ) and NNPDF2.3 [56] for NLO ( $\alpha_s(M_Z) = 0.118$ ) if not mentioned otherwise.

<sup>6</sup> To download NJET visit the project home page at <https://bitbucket.org/njet/njet/>.

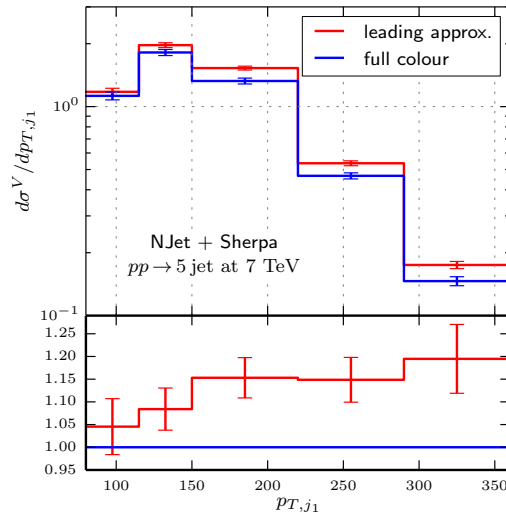


Figure 1: Full colour and leading approximation (as explained in the text) for the virtual corrections to the transverse momentum of the 1st jet in  $pp \rightarrow 5j$ .

Generated events are stored in ROOT Ntuple format [57] which allows for flexible analysis. Renormalization and factorization scales can be changed at the analysis level as well as the PDF set. This technique makes it possible to do extended analysis of PDF uncertainties and scale dependence, which would otherwise be prohibitively expensive for such high multiplicity processes.

### 2.1. Numerical results

Using the above setup we obtain for the 5-jet cross section at 7 TeV

$$\sigma_5^{7\text{TeV-LO}}(\mu = \hat{H}_T/2) = 0.699(0.004)_{-0.280}^{+0.530} \text{ nb}, \quad (4)$$

$$\sigma_5^{7\text{TeV-NLO}}(\mu = \hat{H}_T/2) = 0.544(0.016)_{-0.177}^{+0.0} \text{ nb}. \quad (5)$$

In parentheses we quote the uncertainty due to the numerical integration. The theoretical uncertainty has been estimated from scale variations over the range  $\mu \in [\hat{H}_T/4, \hat{H}_T]$  and is indicated by the sub- and superscripts. As seen in Fig. 2 the total cross section at the scale  $\mu = \hat{H}_T$  is lower than the central value which is the origin of the zero value of the upper error bound. The total cross section at this scale is  $\sigma_5^{7\text{TeV-NLO}}(\mu = \hat{H}_T) = 0.544(0.016)$  nb. For a centre-of-mass energy of 8 TeV the results read:

$$\sigma_5^{8\text{TeV-LO}}(\mu = \hat{H}_T/2) = 1.044(0.006)_{-0.413}^{+0.770} \text{ nb}, \quad (6)$$

$$\sigma_5^{8\text{TeV-NLO}}(\mu = \hat{H}_T/2) = 0.790(0.021)_{-0.313}^{+0.0} \text{ nb}, \quad (7)$$

where we have found  $\sigma_5^{8\text{TeV-NLO}}(\mu = \hat{H}_T) = 0.723(0.011)$  nb.

As usual for a next-to-leading order correction a significant reduction of the scale uncertainty can be observed. In Fig. 2 the scale dependence of the LO and NLO cross section is illustrated. The dashed black line indicates the central scale  $\mu = \hat{H}_T/2$ . The horizontal bands show the cross section uncertainty estimated by a scale variation within  $\mu \in [\hat{H}_T/4, \hat{H}_T]$ .

By comparing Figs. 2a and 2b we observe that a significant part of the NLO corrections comes from using NLO PDFs with the corresponding  $\alpha_s$ . Similar to what has been found in Ref. [10] we conclude that using the NLO PDFs in the LO predictions gives a better approximation to the full result compared to using LO PDFs.

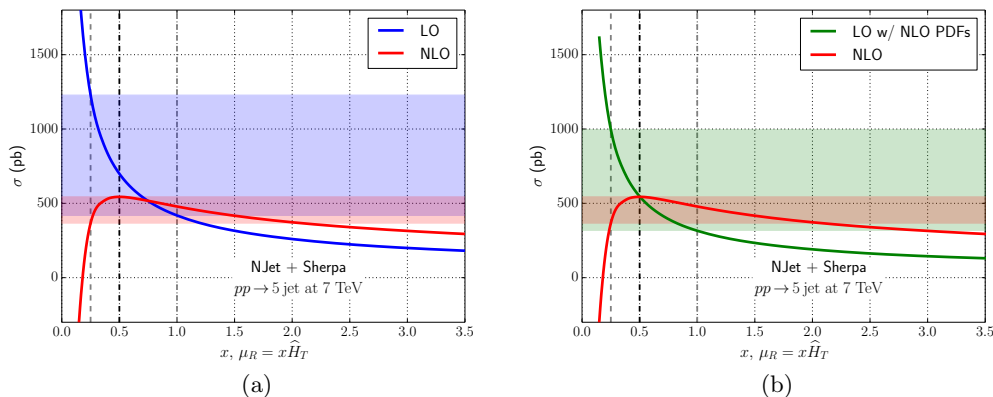


Figure 2: Residual scale dependence of the 5-jet cross section in leading and next-to-leading order using LO (a) and NLO (b) PDFs for LO prediction.

In Tab. 1 we show for completeness the cross sections for two, three and four-jet production as calculated with NJET using the same setup as in the five jet case.

Table 1: Cross sections for 2, 3 and 4 jets at 7 TeV.

| $n$ | $\sigma_n^{7\text{TeV-LO}}$ [nb] | $\sigma_n^{7\text{TeV-NLO}}$ [nb] |
|-----|----------------------------------|-----------------------------------|
| 2   | $768.0(0.9)_{-151.3}^{+203.0}$   | $1175(3)_{-129}^{+120}$           |
| 3   | $71.1(0.1)_{-20.0}^{+31.5}$      | $52.5(0.3)_{-19.3}^{+1.9}$        |
| 4   | $7.23(0.02)_{-2.50}^{+4.37}$     | $5.65(0.07)_{-1.93}^{+0}$         |

The jet rates have been measured recently by ATLAS using the 7 TeV data set [1]. In Fig. 3a we show the data together with the theoretical predictions in leading and next-to-leading order. In case of the six jet rate only LO results are shown. In the lower plot the ratio of theoretical predictions with respect to data is given. With exception of the two-jet cross section the inclusion of the NLO results improves significantly the agreement with data.

In addition to inclusive cross sections it is useful to consider their ratios since many theoretical and experimental uncertainties may cancel between numerator and denominator. In particular we consider

$$\mathcal{R}_n = \frac{\sigma_{(n+1)\text{-jet}}}{\sigma_{n\text{-jet}}}. \quad (8)$$

This quantity is in leading order proportional to the QCD coupling  $\alpha_s$  and can be used to determine the value of  $\alpha_s$  from jet rates. In Fig. 3b we show QCD predictions in NLO using different PDF sets together with the results from ATLAS. The results obtained from NNPDF2.3 are also collected in Tab. 2 where, in addition, the ratios at leading order (using the LO setup with NNPDF2.1) are shown. In case of  $\mathcal{R}_3$  and  $\mathcal{R}_4$  perturbation theory seems to provide stable results. The leading order and next-to-leading order values differ by less than 10%. In addition NNPDF [56], CT10 [58] and MSTW08 [59] give compatible predictions. ABM11 [60] gives slightly smaller results for  $\mathcal{R}_3$  and  $\mathcal{R}_4$ . Within uncertainties the predictions also agree with the ATLAS measurements. The poor description of  $\mathcal{R}_2$  can be attributed to the inclusive two-jet cross section which seems to be inadequately described by a fixed order NLO calculation. As a

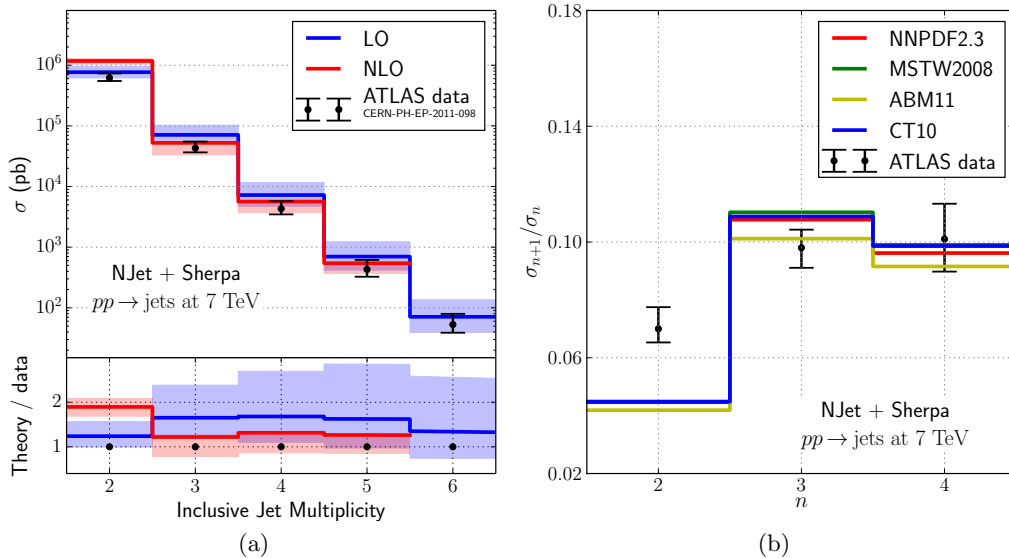


Figure 3: (a) LO and NLO cross sections for jet production calculated with NJET as well as results from ATLAS measurements [1]. (b) NLO NJET predictions with different PDF sets for the jet ratios  $\mathcal{R}_n$  compared with recent ATLAS measurements [1].

Table 2: Results for the jet ratios  $\mathcal{R}_n$  for the central scale of  $\hat{H}_T/2$  and NNPDF2.3 PDF set.

| $\mathcal{R}_n$ | ATLAS [1]                 | LO             | NLO            |
|-----------------|---------------------------|----------------|----------------|
| 2               | $0.070^{+0.007}_{-0.005}$ | 0.0925(0.0002) | 0.0447(0.0003) |
| 3               | $0.098^{+0.006}_{-0.007}$ | 0.102(0.000)   | 0.108(0.002)   |
| 4               | $0.101^{+0.012}_{-0.011}$ | 0.097(0.001)   | 0.096(0.003)   |
| 5               | $0.123^{+0.028}_{-0.027}$ | 0.102(0.001)   | --             |

function of the leading jet  $p_T$ , all PDF sets agree well with the 3/2 ratio ATLAS data at large  $p_T$  as shown in Fig. 4a. In Fig. 4b we compare LO and NLO predictions for  $\mathcal{R}_n$  as function of the leading jet  $p_T$ . While for  $\mathcal{R}_3$  and  $\mathcal{R}_4$  the corrections are moderate for all values of  $p_T$  we observe large negative corrections independent from  $p_T$  in case of  $\mathcal{R}_2$ .

In Fig. 5 we show the transverse momentum and rapidity distributions of the leading jet for five-jet production. Similarly to total cross section we observe significant reduction of the scale uncertainty when going from LO to NLO. Using again the NLO setup to calculate the LO predictions, the NLO calculation gives very small corrections. Over a wide range the LO predictions are modified by less than 10%. A remarkable feature observed already in the 4-jet calculation [9, 10] is the almost constant K-factor.

### 3. Conclusions

In this contribution we have presented first results for five-jet production at NLO accuracy in QCD. We find moderate corrections of the order of 10% at NLO with respect to a leading order computation using NLO PDFs. We have compared theoretical predictions for inclusive jet cross sections and jet rates with data from ATLAS. With the exception of quantities affected by the two-jet rate we find good agreement between theory and data.

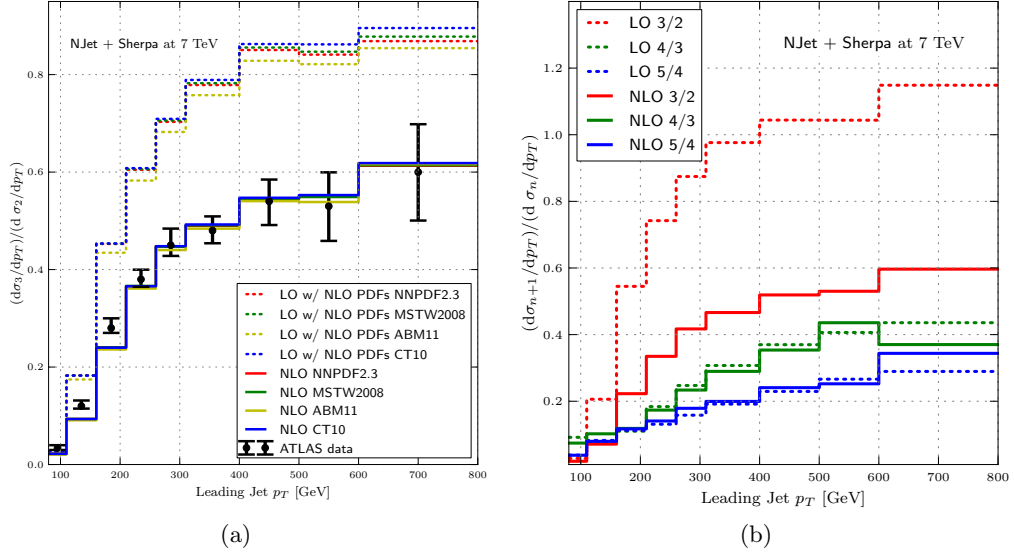


Figure 4: (a) The 3/2 jet ratio as a function of the  $p_T$  of the leading jet compared with ATLAS data [1] ( $R = 0.6$ ). (b) The  $\mathcal{R}_n$  ratios as functions of the  $p_T$  of the leading jet ( $R = 0.4$ ).

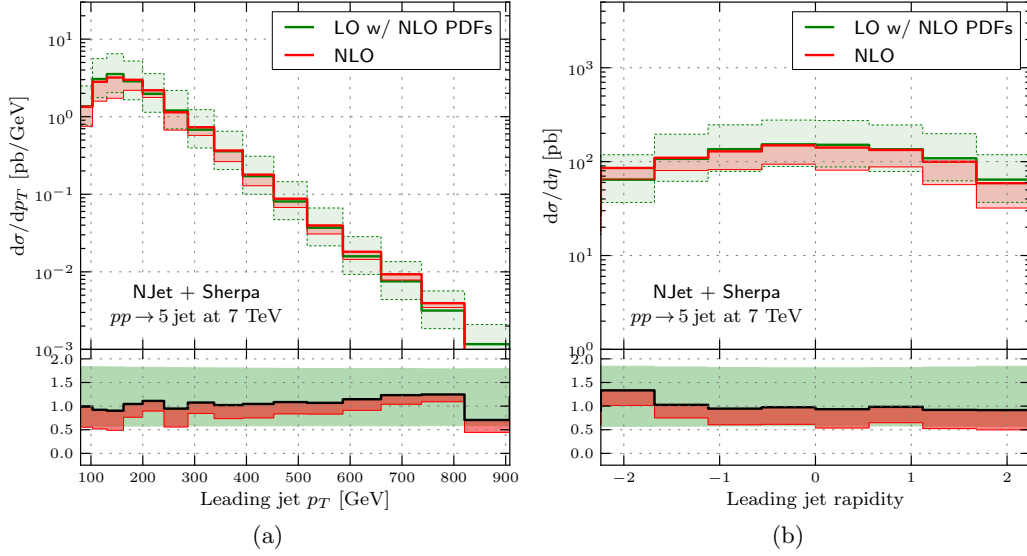


Figure 5: The  $p_T$  and rapidity distributions of the leading jet. Both LO and NLO use the NNPDF2.3 PDF set with  $\alpha_s(M_Z) = 0.118$

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