Three-loop beta-functions and anomalous dimensions in the Standard Model

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Abstract. In this talk the methods and computer tools which were used in our recent calculation of the three-loop Standard Model renormalization group coefficients are discussed. A brief review of the techniques based on special features of dimensional regularization and minimal subtraction schemes is given. Our treatment of γ_5 is presented in some details. In addition, for a reasonable set of initial parameters the numerical estimates of the obtained three-loop contributions are presented.

The renormalization group (RG) proves to be a useful and powerful tool in studying high-energy behavior of the Standard Model. Before the discovery of the Higgs boson RG equations (RGE) were used, among other things, to bound the value of the Higgs self-coupling. However, the bounds significantly depend on the scale at which one expects the appearance of New Physics. The observation of the Higgs boson in 2012 [1, 2] in some sense finalizes the SM since the information about the values of all SM couplings become available from experiments.

Due to this fact, the interest to RG studies of the Standard Model arises again, but at a new level of precision. One- and two-loop results for SM beta-functions have been known for quite a long time [3] and are summarized in Ref. [4].

The first paper with full three-loop calculation of gauge coupling beta-functions within the SM was published in Ref. [5]. The next step was carried out by another group from Karlsruhe [6], which considered, in the specific limit, the three-loop beta-functions for the top quark Yukawa coupling, the Higgs self-coupling and mass parameter. At this state of things our group entered the game. We not only confirmed the results of Refs. [5, 6, 7] but also provided the three-loop expressions for beta-functions of all the Yukawa couplings [8] corresponding to the fermions of third generation. Contrary to K. Chetyrkin and M. Zoller [6], we include the dependence on electroweak couplings.

In the beginning of 2013 our group started the calculation of missing three-loop terms in beta-functions of the Higgs potential parameters. It turns out that the same problem was considered by the authors from Karlsruhe. Unfortunately to us, they managed to obtain and make their results [9] public a week earlier than our group [10]. Nevertheless, in such a complicated calculation it is important to have a confirmation from an independent source. In what follows, we are going to discuss the peculiarities of the procedure used to obtain the results published by our group in a series of papers [8, 10, 11].

First of all, let us mention that the calculation of the three-loop SM beta-functions requires evaluation of millions of Feynman diagrams. This task definitely requires automatization by means of a computer. Fortunately, all necessary tools were available on the market, so we only needed to combine them in a proper way.

It is due to nice features of dimensional regularization and minimal subtraction scheme one can significantly simplify the calculation. Since in the $\overline{\text{MS}}$-scheme all renormalization constants can be extracted from the ultraviolet (UV) divergent parts of the corresponding Green functions, one can modify infra-red (IR) structure of the model to simplify the calculation of counter-terms. This is the essence of the so-called infrared-rearrangement (IRR) trick, which was originally proposed by A.A Vladimirov[12]. This kind of modifications can lead to a spurious IR divergences which should be removed consistently by the so-called R^* operation [13]. However, in many practical cases one can avoid this kind of complications. In our series of paper we made use of two variants of (naive) IRR procedure.

For the calculation of gauge and Yukawa coupling beta-functions it is possible to convert all required two- and three- point Green functions to the massless propagator-type Feynman integrals. It is done via neglecting all internal masses and setting the Higgs boson external momenta entering Yukawa vertex to zero. The evaluation of massless three-loop propagators is performed via a FORM [14] package `MINCER` [15]. This kind of manipulations does not introduce spurious IR divergences. Moreover, since we are interesting only in UV counter-terms it is possible to work from the very beginning within the "unbroken phase" of the SM, in which all fields are massless and the Higgs doublet Φ does not have a vacuum expectation value.

The second approach to IRR, which was used in calculation of Higgs potential parameter beta-functions, is the introduction of an auxiliary mass parameter M in every propagator via iterative application of the following formula [16, 17]

$$\frac{1}{(q-p)^2} = \frac{1}{q^2 - M^2} + \frac{2qp - p^2 - M^2}{q^2 - M^2} \times \frac{1}{(q-p)^2} \tag{1}$$

where q and p are linear combinations of internal and external momenta, respectively. It is clear that if one applies this kind of decomposition a sufficient number of times the last term can be neglected in the calculation of UV divergences (after subtraction of subdivergences). It turns out that for the scalar four-point Green functions considered only the first term in Eq. (1) is necessary. Consequently, we are left with massive vacuum integrals, which can be calculated by either public `MATAD` package [18] or private `BAMBA` code written by V. Velizhanin. In such an approach no spurious IR divergences appear so it can be used in the situations when a naive application of the first variant of IRR fails. However, the price to pay for this advantage is the necessity of explicit calculation of diagrams with counter-term insertions. This is due to the fact that one needs to introduce counter-terms for divergences contributing to the auxiliary masses for vector and scalar bosons. For further details see Ref. [16].

It is worth mentioning that we can still exploit the symmetries of the unbroken SM. For example, all components of the Higgs boson doublet should have the same auxiliary mass counter-term. The same is true for the SU(2) gauge bosons.

Moreover, as it is stated in Ref. [17], the auxiliary mass appearing in the numerator in RHS of Eq. (1) can be safely neglected since it can only contribute to the unphysical mass counter-terms. In addition, one can also skip Feynman diagrams with vacuum subdiagrams. In spite of the fact that these subdiagrams are non-zero when the auxiliary mass is introduced, they still can be neglected due to the same reasons.

As it was noticed above there are a lot of diagrams which should be generated and evaluated in order to find three-loop contributions to the considered quantities. In our calculation we made use of two popular codes, `FeynArts`[19] and `DIANA` [20]/`QGRAF` [21], which generate necessary diagrams and produce corresponding analytic expressions. Both packages require a model file prepared in a special format to do their job. Since we wanted to simplify the calculation as much as possible we prepared a model file for the unbroken SM in the background field gauge (BFG) [22, 23]. This kind of gauge allows one to find gauge coupling beta-functions solely from UV-divergences of the corresponding gauge field propagators. We used a very fast `LanHEP` code [24] by A. Semenov to derive all SM vertices from the considered SM Lagrangian (see Ref. [11]) in `FeynArts` notation. It is worth mentioning that the Karlsruhe group made use of alternative package `FeynRules` [25] to solve similar problem. Latter on a simple script was written to convert the `FeynArts` model file to that of `DIANA`.

A typical problem which arises in this kind of calculation is internal momenta identification. In order to evaluate a Feynman diagram one needs to use the momenta notation of the chosen code (`MINCER`/`MATAD`/`BAMBA`). In the case of gauge and Yukawa couplings the problem was solved with the help of routine, `MapMincer`, which associate with every `FeynArts` topology the corresponding `MINCER` topology and distribute `MINCER` momenta accordingly. It is worth mentioning that `MapMincer` can deal with three-point vertices with one external leg carrying zero momentum (i.e., when internal lines have dots). The corresponding routine for `DIANA`/`QGRAF`, `MapDiana`, performs similar task, but maps every generated topology to fully massive vacuum integrals, which appear after the mentioned"exact" decomposition (1) of internal propagators[1].

Given a model file for `FeynArts`/`DIANA` together with the correct mapping of internal momenta to the notation of the utilized three-loop codes it is straightforward to generate and calculate one-, two-, and three-loop contributions to the 1PI Green-functions presented in Fig. 1. For the SU(3) color algebra the `COLOR` [26] package was used.

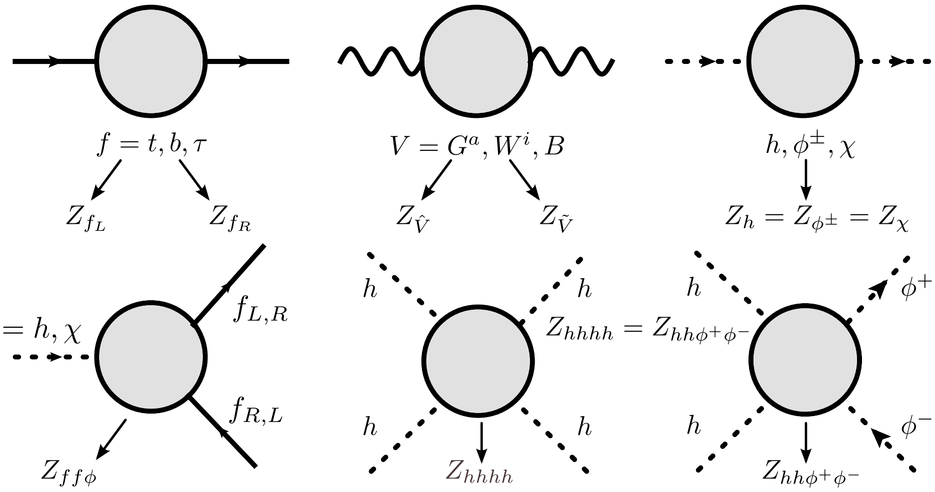

Figure 1: The Green functions, considered in the beta-function calculations, together with the corresponding renormalization constants. Left- and right-handed fermions, denoted by $f_{L,R}$, renormalise differently in the SM. The same is true for background $\hat{V}$ and quantum $\tilde{V}$ gauge fields in BFG employed. The SU(2) invariance implies the presented equalities, which serve as an additional cross-check.

From two-point Green functions we extract the corresponding wave function renormalization constants in the $\overline{\text{MS}}$-scheme. It is worth pointing that the self-energies of both background and quantum gauge fields are considered. The renormalization constants of the former, $Z_{\hat{V}}$, are directly connected to that of gauge couplings and the renormalization of the latter corresponds to the Z-factors of three gauge-fixing parameters, ξ_G, ξ_W, and ξ_B, which we keep non-zero during

[1] Both `MapMincer` and `MapDiana` are coded by A. Pikelner.

the whole stage of calculation. The independence of the final results on these parameters serves as an important cross-check of the obtained expressions.

The three-point 1PI functions for the Yukawa vertices with neutral Higgs bosons h and χ are also presented in Fig. 1. It is interesting to note that our calculation explicitly demonstrated that the semi-naive treatment of γ_5 discussed below is only applicable if one takes into account the gauge anomaly cancellation condition $N_C = 3$. Here N_C denotes the number of colors.

The beta-functions of the SM parameters are extracted from the corresponding renormalization constants. For the gauge and Yukawa couplings we used the following relations

$$Z_{g_1} = Z_{\hat{B}_1}^{-1/2}, \qquad Z_{g_2} = Z_{\hat{W}}^{-1/2}, \qquad Z_{g_s} = Z_{\hat{G}}^{-1/2}, \qquad Z_{y_f} = \frac{Z_{ff\phi}}{\sqrt{Z_{f_L} Z_{f_R} Z_\phi}}, \tag{2}$$

where g_1, g_2 are SU(2) and U(1) gauge couplings, respectively, g_s denotes the strong coupling, and y_f is the Yukawa coupling associated with the (right-handed) fermion $f = t, b, \tau$. Both neutral components, $\phi = h, \chi$, gave the same result, and, thus, provided us with a confirmation of the validity of the obtained expressions. For the Higgs self-coupling it is impossible to use `MINCER` naively, so Feynman diagrams for the four-point functions (see Fig. 1) converted to the fully massive vacuum integrals were calculated with the help of private code `BAMBA` (by V. Velizhanin, who considered the $hh\phi^+\phi^-$ vertex) and public package `MATAD` (by A. Bednyakov and A. Pikelner who considered the fully symmetric $hhhh$ vertex). These two independent evaluations lead to the same final expression for the vertex renormalization constants, i.e., confirming the SU(2) relation $Z_{hhhh} = Z_{hh\phi^+\phi^-}$.

A comment on the Higgs mass parameter m^2 is in order. It is possible to obtain the corresponding anomalous dimension by considering the renormalization of the $|\Phi|^2$ composite operator within the unbroken(=massless) SM (see, e.g., Ref. [6]). This kind of result can be found at almost no cost from the calculation of $hh\phi^+\phi^-$ vertex. It is sufficient to select the diagrams, which have ϕ^+ and ϕ^- external fields connected to the same four-point vertex (see Fig. 2), and weight different contributions with a correct combinatorial factor. This restricted set of diagrams gives rise to the $Z_{hh[\phi^+\phi^-]}$ renormalization constant.

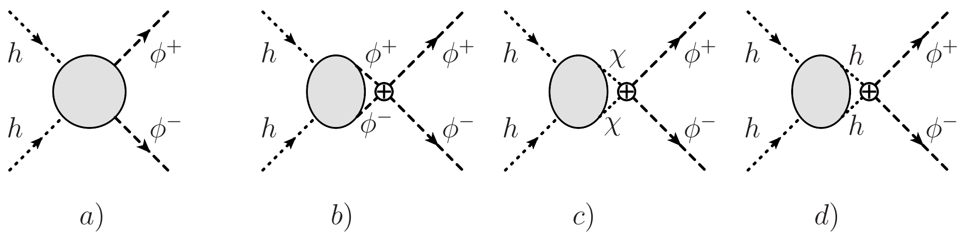

Figure 2: The restricted set of Feynman diagrams which is used to obtain the renormalization constant $Z_{hh[\phi^+\phi^-]}$. The diagrams of the first type have to be multiplied by 1/2.

At the end of the day, the renormalization constants for λ and m^2 are obtained with the help of the following relations

$$Z_\lambda = \frac{Z_{hhhh}}{Z_h^2} = \frac{Z_{hh\phi^+\phi^-}}{Z_h Z_{\phi^\pm}}, \qquad Z_{m^2} = \frac{Z_{hh[\phi^+\phi^-]}}{Z_h} \tag{3}$$

From renormalization constants Z_{a_i} for the dimensionless SM parameters,

$$a_i = \left(\frac{5}{3}\frac{g_1^2}{16\pi^2}, \frac{g_2^2}{16\pi^2}, \frac{g_s^2}{16\pi^2}, \frac{y_t^2}{16\pi^2}, \frac{y_b^2}{16\pi^2}, \frac{y_\tau^2}{16\pi^2}, \frac{\lambda}{16\pi^2} \right), \tag{4}$$

it is straightforward to obtain the corresponding beta-functions

$$\beta_i(a_k) = \frac{da_i(\mu,\epsilon)}{d\ln\mu^2}\bigg|_{\epsilon=0}, \quad \beta_i = \sum_l a_l \frac{\partial c_i^{(1)}}{\partial a_l} - c_i^{(1)}, \quad a_{i,\text{Bare}}\mu^{-2\epsilon} = Z_{a_i} a_i = a_i + \sum_{n=1}^{\infty} \frac{c_i^{(n)}}{\epsilon^n} \tag{5}$$

Here μ is the $\overline{\text{MS}}$ renormalization scale, $\epsilon = (4-D)/2$ is the parameter of dimensional regularization, and c_i denotes the coefficient for the single pole in ϵ in the expression for Z_{a_i}, which enters the relation between the bare parameters $a_{i,\text{Bare}}$ and the renormalized ones.

For the anomalous dimension of the Higgs mass parameter m^2 one can use similar formulae

$$\gamma_{m^2}(a_k) = \frac{d\ln m^2}{d\ln\mu^2}\bigg|_{\epsilon=0} = \sum_l a_l \frac{\partial c_{m^2}^{(1)}}{\partial a_l}, \qquad m^2_{\text{Bare}} = Z_{m^2} m^2 = m^2\left(1 + \sum_{n=1}^{\infty}\frac{c_{m^2}^{(n)}}{\epsilon^n}\right) \tag{6}$$

The full analytical results for the considered quantities can be found in ancillary files of the arXiv versions of our papers. The intermediate expressions, e.g., all renormalization constants, can also be obtained, if needed, from the authors. It is worth mentioning that the beta-functions of all fundamental SM parameters are free from gauge–parameter dependence, which is a crucial test for our calculation. In addition, the anomalous dimension of the Higgs doublet can be of some interest, so we also include the corresponding expression in the ancillary files of Ref. [10].

Before going to the results we would like to touch on the issue related to the definition of the γ_5 matrix within the dimensional regularization. It is known from the literature (see, e.g., Ref. [27] and recent explicit calculation [6]) that the traces with an odd number of γ_5 appearing for the first time in the three-loop diagrams require special treatment. We closely follow the semi-naive approach presented in Refs. [6, 7]. First of all, we anticommute γ_5 to the rightmost position in a fermion chain and use $\gamma_5^2 = 1$. In the "even" traces all γ_5 are contracted with each other, so the corresponding expressions can be treated naively in dimensional regularization. In "odd" traces we are left with only one γ_5. These traces are evaluated as in four dimensions and produce totally antisymmetric tensors via the relation

$$\text{tr}\,(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5) = -4i\epsilon^{\mu\nu\rho\sigma} \tag{7}$$

with $\epsilon^{0123} = 1$.

Due to the fact that we use both the γ_5 anticommutativity and the four-dimensional relation (7), the cyclicity of the trace should be relinquished [28]. One has to choose a certain "reading prescription" for an "odd" Dirac trace, i.e., start reading a closed fermion chain from a proper place, in order to achieve the correct final result. However, in our calculations the problem of γ_5 positioning within the "odd" traces is solved implicitly since the diagram generation routines split the traces for us at certain points. It should be stressed that a non-trivial contribution to the considered quantities can only appear when there are two "odd" traces in a diagram, since two ϵ-tensors should be "contracted" with each other to produce a non-zero effect. We can distinguish two situations. Two "odd" traces in our three-loop calculations can appear either as two internal fermion loops in a diagram with external bosons or, if one considers Green functions with two external fermions, one Dirac trace from internal fermion loop can be combined with the trace appearing after contraction with an appropriate projector.

It is easy to convince oneself that in the case of two internal traces only triangle subloops with three external vector particles can potentially produce "eps"-tensors. However, it is known that in the SM these kind of traces cancel upon summation over all fermion species due to the absence of gauge anomalies [29, 30]. The same is true if one considers fermion self-energies up to three-loops.

It turns out that the non-trivial contribution due to the contraction of two ϵ-tensor appears from the Yukawa vertex (see Fig. 3). This kind of diagrams was also considered in Ref. [6].

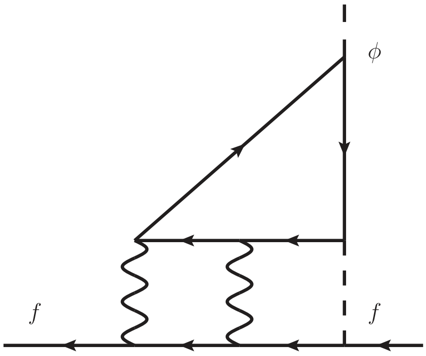

Figure 3: The diagrams for the fermion-fermion-Higgs vertex ($ff\phi$), which produce a non-zero contribution due to contraction of two ϵ-tensors appearing from two Dirac traces with γ_5. The ambiguity in positioning of γ_5 within the traces does not affect the UV-divergent part.

A final remark about γ_5 is again related to the fact that the ambiguity in the choice of "reading" point in the "odd" traces can only spoil our result for the three-loop UV-divergence in the case of the mentioned triangle subgraphs, for which one can set all "odd" traces to zero from the very beginning due to anomaly cancellations. For the only non-trivial case all "reading" points are equivalent since the difference gives the contribution $\mathcal{O}(\epsilon^0)$ which we neglect here.

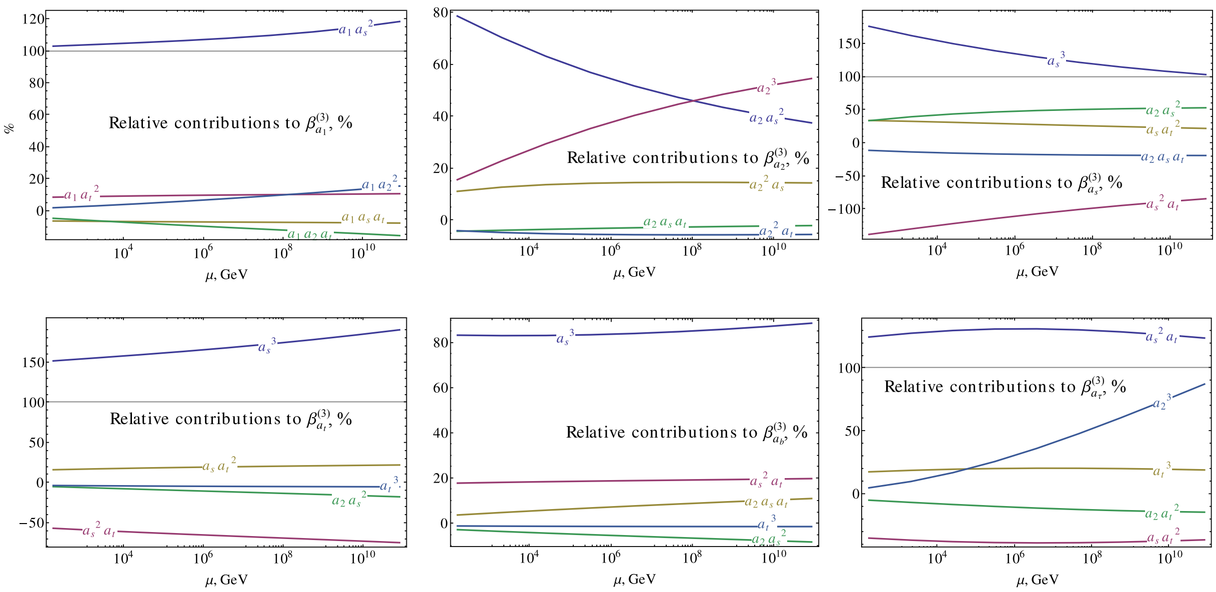

Figure 4: The scale dependence of the relative contributions of different terms in the three-loop corrections to the beta-functions of the (squared) gauge and Yukawa couplings. Only the most sizable corrections are shown. The boundary conditions at scale $\mu = 100$ GeV are given in (8).

It is obvious, that the resulting expression for the three-loop contributions to the considered renormalization group quantities are too lengthy to be presented here. For the demonstration purposes only, we would like to show how different relative contributions to the three-loop corrections to the SM coupling beta-functions evolve with renormalization scale μ (see Fig. 4). For a reasonable choice of the SM initial running parameters[2] at the scale $\mu = 100$ GeV

$$g_1 = 0.3576,\ g_2 = 0.6514,\ g_s = 1.2063,\ y_t = 0.9665,\ y_b = 0.016,\ y_\tau = 0.01,\ \lambda = 0.13 \qquad (8)$$

[2] obtained with the help of F. Bezrukov code http://www.inr.ac.ru/~fedor/SM

we solve the corresponding RGE numerically up $5 \cdot 10^{10}$ GeV. With the help of these solutions one can evaluate the three-loop beta-functions at any scale and find how different terms contribute to the total value of $\beta_i^{(3)}$. From Fig. 4 one can see that the dominant contributions is due to the strong and top Yukawa couplings. However, with the increase of the renormalization scale the SU(2) coupling can also play a role.

It is fair to say that the most interesting SM beta-function is that of the Higgs self-coupling, since from the evolution of the latter one can deduce the so-called "vacuum stability bound" (see recent papers [31, 32, 33, 35] and references therein). In Fig. 5 one can find the same evolution of the relative contributions to $\beta_\lambda^{(3)}$. In addition, the slice (λ, β_λ) of the phase space (a_i, β_{a_i}) is presented together with trajectories, obtained with the help of one-, two-, and three-loop evolution from 100 GeV to $5 \cdot 10^{10}$ GeV. One can see that for the given set of initial conditions the running $\lambda(\mu)$ is driven to zero faster when one-loop RGEs are employed instead of two- or three-loop ones. The difference between two- and three-loop evolution is not sizable, but the fact that with three-loop corrections the scale at which λ hits zero slightly higher, favours the latter.

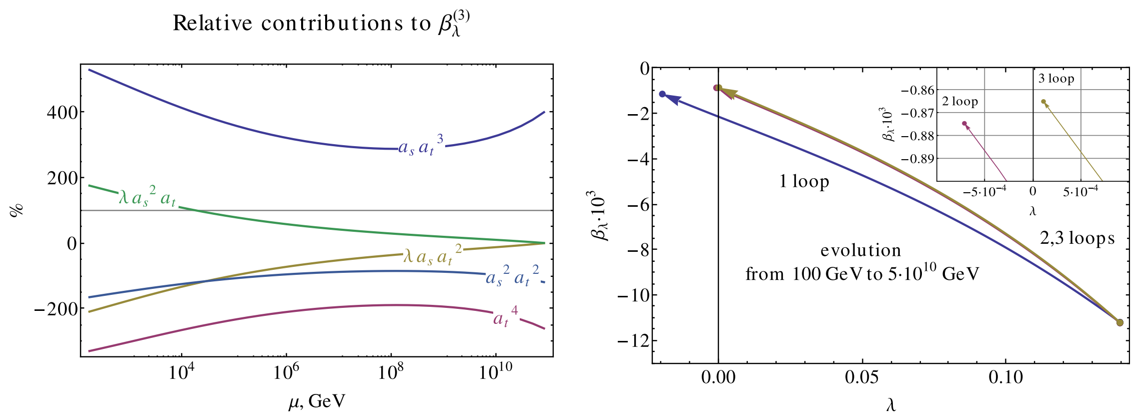

Figure 5: The scale dependence of the relative contributions to the three-loop corrections $\beta_\lambda^{(3)}$. Only the most sizable corrections are shown. The phase space trajectories from 1-,2-,and 3-loop evolution are provided in the plane (λ, β_λ). The corresponding boundary conditions are given in Eq. (8)

To conclude, we obtained the three-loop beta-functions for the SM parameters. The results for gauge and Higgs-potential couplings coincide with that obtained by two Karlsruhe groups. The beta-functions for Yukawa couplings were obtained for the first time. Moreover, we established a framework that allow us to carry out a similar calculation within an "arbitrary" QFT model. However, it should be stressed that in a self-consistent RG analysis of the chosen model the obtained RGEs should be accompanied by the so-called threshold (matching) corrections (see, e.g., Refs. [31, 34, 32, 35] for the recent SM results).

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