Automatic calculation in quarkonium physics

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Abstract. In this report, an automatic calculating package based on REDUCE and RLISP, FDC, is introduced, especially its one-loop calculation part and its special treatment for quarkonium physics. With FDC, many works have been completed, most of them are very important in solve/clarify current puzzles in quarkonium physics.

1. FDC package and its progress

FDC, which stands for Feynman Diagram Calculation, is a package developed to do automatic calculation for physics processes, and it contained four parts: 1) prepare Lagrangian and deduce Feynman rules for first principle model, 2) generate all possible Feynman diagrams for the given process in a given model; 3) manipulate the amplitude of the process analytically; 4) generate FORTRAN code for final numerical results. It was started about twenty year ago [1] and written in REDUCE and RLISP to generate FORTRAN Code. First version of FDC has been presented at AIHENP93 [2]. FDC-MSSM part [3] was developed to automatic construct Lagrangian and deduce Feynman rules for supersymmetry model around 2000. FDC-PWA, a Partial Wave Analysis application for experiments, had been developed until 2002. A brief introduction on these early tree-level achievement could be found in Ref. [4]. In the following, loop-level achievement will be introduced since 2002.

1.1. One loop calculation in FDC

With the development of experiment in high energy physics, accuracy of the measurement has been gradually improved. This makes it more and more important to include higher order corrections in theoretical predictions. As an automatic calculating system, FDC has to be capable of one-loop calculation. The development of one-loop calculation part in FDC started from 2002, and finished in 2007. Thereafter, a few improvements have been introduced.

1.1.1. Real corrections A two-cutoff phase space slicing method [5] is realized in FDC to deal with IR divergence in real corrections. Two cutoffs, δ_s and δ_c are introduced to separate the phase space into three parts: soft, hard-collinear, hard-noncollinear. The hard-noncollinear part is finite, and can be calculated numerically with traditional Monte-Carlo method. Both the soft and hard-collinear part are factorized in soft/collinear limit, and added to corresponding virtual correction processes. All the divergence (including those in virtual corrections) are separated analytically, and then sum up to check if they are really cancelled with each other.

1.1.2. Virtual corrections First, with the input of renormalization constant, counter term diagrams will be generated automatically in FDC. There are two ways in FDC to generate square of amplitude: square the amplitude analytically; generate numerical result (in FORTRAN Code) of amplitude, then square it. These two ways will lead to different tensor reduction and then bring a cross-check inside FDC. Besides, all the divergence (both UV and IR) are separated during the calculation of amplitude analytically. Loop integrals are treated via following steps:

- For one and two point integrals, Feynman parametrization is used directly to derive the analytical results.
- Besides above, Passarino-Veltman reduction method is used to reduce tensors into scalars.
- Some other relations are used in the reduction of $N \geq 5$ point integrals.
- A special $i\epsilon$-regularization scheme is used to derive scalars under dimensional regularization.

1.1.3. $i\epsilon$-regularization scheme An N-point scalar integral under D-dimension regularization can be defined as

$$T_0^{(N)}(p_1, \ldots, p_{N-1}, m_0, m_1, \ldots, m_{N-1}) = \mu^{4-D} \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{1}{N_0 \ldots N_{N-1}}, \tag{1}$$

where $N_n = (q + p_n)^2 - m_n^2 + i\epsilon, \quad n = 0, \ldots, N-1$ are the propagators.

According to Ref. [6], the IR singularities part of the scalar can be expressed as sum of a few 3-point scalars with IR singularities:

$$T_0|_{\text{sing}}^{(N)} = \sum_{n=0}^{N-1} \sum_{\substack{k=0 \\ k \neq n, n+1}}^{N-1} A_{nk} C_0(p_0, \ldots, p_k, m_n, m_{n+1}, m_k). \tag{2}$$

Thus, we can evaluate the scalar integral via

$$T_0^D = T_0^\epsilon - T_0|_{\text{sing}}^\epsilon + T_0|_{\text{sing}}^D, \tag{3}$$

where T_0^ϵ, $T_0|_{\text{sing}}^\epsilon$ are under $i\epsilon$-regularization, and $T_0|_{\text{sing}}^D$ is easily obtained in D-dimension regularization.

Here is the procedure to deal with the integral under $i\epsilon$ regularization:

- keep $i\epsilon$ in the propagators to make the scalar integral well defined.
- let the dimension back to 4.
- do the integration following the standard way described in Ref. [7].
- expand the results in $i\epsilon$.
- This way is suitable to program and is realized in FDC package.

1.1.4. Improvement in loop integrals In 2011, the reduction method for loop integrals proposed in Ref. [8] is implemented in FDC. It has many advantages:

- It can reduce integrals with abnormal dimension and denominators, which may appear e.g. in the calculation of processes involving P-wave states.
- It can reduce tensors with high (4^+) rank and more (5^+) external momenta at same time, where Passarino-Veltman reduction method may fail.
- It can further reduce some integrals, e.g. scalar integrals containing Coulomb singularities in the calculation of quarkonium physics.
- It is easy to obtain coefficients of certain Lorentz term with this method.

1.2. Special treatment for NRQCD

Quarkonium physics has been a hot topic since first quarkonium was found in 1974. As a factorization formalism, nonrelativistic QCD (NRQCD) [9] is now the most common framework to study quarkonium productions and decays. It allows consistent theoretical predictions to be made and improved perturbatively in the QCD coupling constant and the heavy-quark relative velocity. According to NRQCD factorization formalism, production (or decay) of a quarkonium H can be expressed as

$$d\sigma[H] = \sum_n f_n \langle \mathcal{O}_n^H \rangle, \tag{4}$$

where n represents possible $Q\bar{Q}$ intermediate states. f_n is the short-distance coefficient which can be calculated perturbatively, while $\langle \mathcal{O}_n^H \rangle$ is is the production (decay) matrix elements of H, which are fully governed by non-perturbative QCD effects. In order to calculate the short-distance coefficient, two spin projection operators [10]

$$
\begin{aligned}
\Pi_0(P,p) &= \frac{1}{2\sqrt{2}E(E+m)} \left(\frac{1}{2}\slashed{P} + m + \slashed{p} \right) \frac{\slashed{P}+2E}{4E} \gamma_5 \left(\frac{1}{2}\slashed{P} - m - \slashed{p} \right), \\
\Pi_1(P,p,\epsilon) &= \frac{-1}{2\sqrt{2}E(E+m)} \left(\frac{1}{2}\slashed{P} + m + \slashed{p} \right) \frac{\slashed{P}+2E}{4E} \slashed{\epsilon} \left(\frac{1}{2}\slashed{P} - m - \slashed{p} \right),
\end{aligned} \tag{5}
$$

are implemented in FDC for spin singlet and triplet intermediate states, where m is the mass of heavy quark, P is the momentum of quarkonium, $p = (p_Q - p_{\bar{Q}})/2$ is the "relative momentum" between heavy quark pair, and $E \equiv \sqrt{m^2 - p^2}$ can be regarded as half of the "mass" of quarkonium.

2. Applications

Based on above progress in FDC, many important work are completed in quarkonium physics.

2.1. Exclusive J/ψ production at the B factories

2.1.1. $e^+e^- \rightarrow J/\psi + \eta_c$ Experiment data for this process measured at $\sqrt{s} = 10.6$ GeV by Belle [11] is

$$\sigma[J/\psi + \eta_c] \times \mathcal{B}^{\eta_c}[\geq 2] = (25.6 \pm 2.8 \pm 3.4) \text{ fb} \tag{6}$$

and by BABAR [12] is

$$\sigma[J/\psi + \eta_c] \times B^{\eta_c}[\geq 2] = (17.6 \pm 2.8^{+1.5}_{-2.1}) \text{ fb}, \tag{7}$$

where $B^{\eta_c}[\geq 2]$ denotes the branching fraction for the η_c decaying into at least two charged tracks. On the other hand, LO theoretical predictions in NRQCD are about $2.3 \sim 5.5$ fb [13, 14, 15], which is an order of magnitude smaller than the experimental results.

Such a large discrepancy between experimental results and theoretical predictions brings a challenge to the current understanding of charmonium production based on NRQCD. Many studies have been performed in order to resolve the problem. Braaten and Lee [13] have shown that the relativistic corrections would increase the cross section by a factor of about 2, which boost the cross section to 7.4 fb. And the NLO QCD correction of the process has been studied by Zhang *et al.* [16], which can enhance the cross section with a K factor (the ratio of NLO to LO) of about 2 and reduce the large discrepancy. Later the relativistic corrections have been studied again [17, 18], which are significant, and when combined with the NLO QCD corrections, may resolve the large discrepancy.

Due to the complexity and importance of NLO calculation for this process, we performed another independent calculation using FDC and obtained an analytic result [19]. The numerical

Table 1. Cross sections for $e^+e^- \to J/\psi + \eta_c$ with different charm quark mass m_c and renormalization scale μ. $\sqrt{s} = 10.6$ GeV is the center-of-mass energy.

m_c(GeV)	μ	$\alpha_s(\mu)$	σ_{LO}(fb)	σ_{NLO}(fb)	σ_{NLO}/σ_{LO}
1.5	m_c	0.369	16.09	27.51	1.710
1.5	$2m_c$	0.259	7.94	15.68	1.975
1.5	$\sqrt{s}/2$	0.211	5.27	11.14	2.114
1.4	m_c	0.386	19.28	34.92	1.811
1.4	$2m_c$	0.267	9.19	18.84	2.050
1.4	$\sqrt{s}/2$	0.211	5.76	12.61	2.189

results are shown in Table 1, which is consistent with Ref. [16]. Our work confirmed former calculation analytically, and is also another self check of FDC. Recently, this process is revisited by Dong *et al.* by including $\mathcal{O}(\alpha_s v^2)$ corrections [20]. The corrections mildly enhance the NRQCD predictions.

It should also be mentioned that same process has also been studied in another framework. Unlike NRQCD, Braguta *et al.* find that the cross section at LO can be 26.7 fb in the framework of the light cone (please check details in Ref. [21], and also Braguta's talk in this conference [22]).

2.1.2. $e^+e^- \to J/\psi + J/\psi$ LO NRQCD predictions for this channel was given by Bodwin *et al.* [23] in 2003. They found that the cross section of this channel may be larger than that of $J/\psi + \eta_c$ by a factor of 1.8, in spite of a suppression factor α^2/α_s^2. They suggested that a significant part of the discrepancy of $J/\psi + \eta_c$ production may be explained by this process. But new analysis performed by Belle [11] based on a 3 times larger data set in 2004 found no evidence for this channel. This becomes a new puzzle in double charmonia production at the B factories.

NLO QCD corrections and relativistic corrections for the dominant photon-fragmentation contribution diagrams are studied by Bodwin *et al.* [24], and the results show that the cross section is decreased by K factors of 0.39 and 0.78 for the NLO QCD and relativistic corrections respectively. In 2006, a more reliable estimate, 1.69 ± 0.35 fb, was given by Bodwin et al. [25]

Table 2. Cross sections for $e^+e^- \to J/\psi + J/\psi$ with different charm quark mass m_c and renormalization scale μ, and $\sqrt{s} = 10.6$ GeV.

m_c(GeV)	μ	$\alpha_s(\mu)$	σ_{LO}(fb)	σ_{NLO}(fb)	σ_{NLO}/σ_{LO}
1.5	m_c	0.369	7.409	-2.327	-0.314
1.5	$2m_c$	0.259	7.409	0.570	0.077
1.5	$\sqrt{s}/2$	0.211	7.409	1.836	0.248
1.4	m_c	0.386	9.137	-3.350	-0.367
1.4	$2m_c$	0.267	9.137	0.517	0.057
1.4	$\sqrt{s}/2$	0.211	9.137	2.312	0.253

With the help of FDC, we perform a full NLO QCD calculation to this process [26]. As shown in Table 2, the cross section would be much smaller than the rough estimate in Ref. [24].

Therefore it is easy to understand why there was no evidence for the process $e^+e^- \to J/\psi + J/\psi$ at B factories.

2.2. Inclusive J/ψ production at the B factories

The cross section for inclusive J/ψ production in e^+e^- annihilation was measured by BABAR [27, 12] as $2.54 \pm 0.21 \pm 0.21$ pb and Belle [28, 29] as $1.45 \pm 0.10 \pm 0.13$ pb. These measurements include both $J/\psi + c\bar{c} + X$ and $J/\psi + X_{\mathrm{non-}c\bar{c}}$ parts in the final states. Many theoretical studies [30, 31, 32, 33, 34, 14, 35, 36, 37] have been performed on this production at LO in NRQCD and the results for inclusive J/ψ production cover the range $0.6 \sim 1.7$ pb depending on parameter choices. A further analysis by Belle [29] gives

$$\sigma(e^+e^- \to J/\psi c\bar{c} + X) = 0.87^{+0.21}_{-0.19} \pm 0.17 \text{ pb}. \tag{8}$$

It is about 5 times larger than the LO NRQCD prediction [14]. However, this large discrepancy was partially resolved by considering both NLO correction and feed-down from higher excited states [38].

2.2.1. $J/\psi + X_{\mathrm{non-}c\bar{c}}$ The above measurements infer that $\sigma[e^+e^- \to J/\psi + X_{\mathrm{non-}c\bar{c}}] = 0.6$ pb. For this part, the contributions from the color-singlet (CS) and color-octet (CO) channels are about 0.2 pb and 0.27 pb, respectively, at the LO in NRQCD [37]. However, the signal of the CO contribution was not found in the experiment [27, 28]. Therefore, the experimental measurement by Belle is about 3 times larger than the theoretical prediction from the CS channel at LO, and can be much more than 3 times by BABAR. To achieve a reasonable theoretical prediction for this channel, we perform a NLO QCD calculation in Ref. [39].

Table 3. Cross sections for $J/\psi + X_{\mathrm{non-}c\bar{c}}$ part with different charm quark mass m_c where the renormalization scale $\mu = 2m_c$ and $\sqrt{s} = 10.6$ GeV.

| m_c(GeV)	$\alpha_s(\mu)$	$\sigma^{(0)}$(pb)	$a(\hat{s})$	$\sigma^{(1)}$(pb)	$\sigma^{(1)}/\sigma^{(0)}$
1.4	0.267	0.341	2.35	0.409	1.20
1.5	0.259	0.308	2.57	0.373	1.21
1.6	0.252	0.279	2.89	0.344	1.23

The cross section at NLO can be expressed as

$$\sigma^{(1)} = \sigma^{(0)} \left\{ 1 + \frac{\alpha_s(\mu)}{\pi} \left[a(\hat{s}) + \beta_0 \ln\left(\frac{\mu}{2m_c}\right) \right] \right\}, \tag{9}$$

where β_0 is the one-loop coefficient of the QCD beta function, and $\hat{s} = s/(2m_c)^2$ is a dimensionless kinematic variable. The numerical results can be found in Table 3. And the scale dependence of the cross section is shown in Fig. 1 and it is improved significantly at NLO. Our final numerical result can be expressed as

$$\sigma^{(1)} = 0.373^{+0.036}_{-0.079} \text{ pb} \tag{10}$$

where the theoretical uncertainty is from the choices of m_c and μ, with $m_c = 1.4$ GeV and $\mu = 2m_c$ for the upper boundary and $m_c = 1.6$ GeV and $\mu = \sqrt{s}/2$ for the lower boundary.
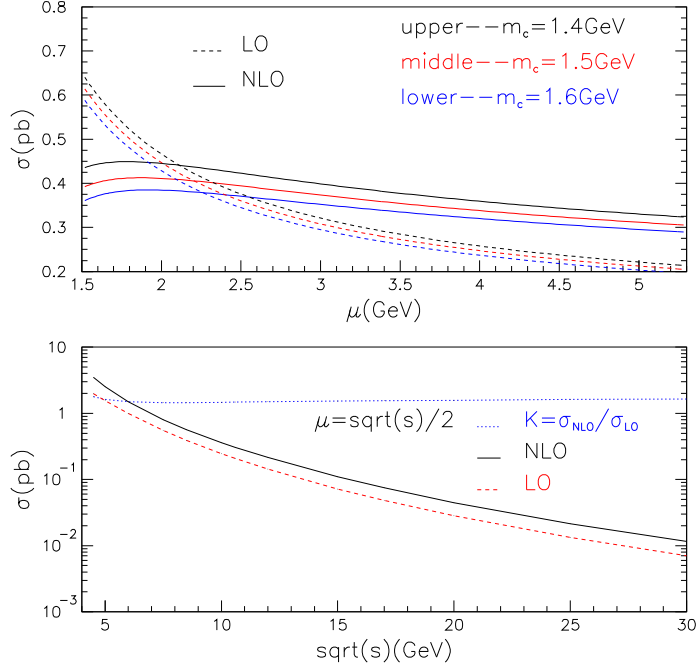
Figure 1. Cross sections for $J/\psi + X_{\text{non}-c\bar{c}}$ as function of the renormalization scale μ and the center-of-mass energy $\sqrt{s}$.

It should be mentioned that same work is also finished by Ma *et al.* [40] almost at the same time. Our results are in agreement with theirs. Also, most recent experimental measurement for this part is given by Belle[41] as

$$\sigma[e^+e^- \to J/\psi + X_{\text{non}-c\bar{c}}] = 0.43 \pm 0.09 \pm 0.09 \text{ pb}. \tag{11}$$

It seems that after including the NLO QCD corrections and feed-down contribution from higher charmonium states, the data can be almost saturated by CS channel, and leaves little room for CO contribution.

2.2.2. $J/\psi + c\bar{c}$ As mentioned above, the discrepancy between experimental data and theoretical predictions of this channel was partially resolved by considering both NLO correction and feed-down from higher excited states [38]. And more recent experimental measurement [41] gives

$$\sigma[e^+e^- \to J/\psi + c\bar{c}] = 0.74 \pm 0.08^{+0.09}_{-0.08} \text{ pb}, \tag{12}$$

which is even closer to the theoretical predictions. Since the calculation of the NLO QCD correction to this process is quite complicated and is very important in explaining the experimental data, it is desirable to have an independent calculation. A more important point is that there are already the momentum and angular distributions for J/ψ production in the new measurement. Therefore, we perform another study on the NLO QCD correction to $e^+e^- \to J/\psi + c\bar{c}$ with our FDC package [42].

Our results for the total cross section are shown in Table. 4, they are in agreement with the previous result in Ref. [38] when their renormalization scheme and input parameters are used. The scale dependence of this channel should be considered as scale dependence is still significant in this part. One possible way, although debatable, to choose the renormalization scale is the BLM scale [43]. A unique scale choice μ^* is obtained and the cross section at NLO is expressed as

$$\sigma^{(1)} = \sigma^{(0)}(\mu^*)[1 + \frac{\alpha_s(\mu^*)}{\pi}b(\hat{s})]. \tag{13}$$

Table 4. Cross sections for $J/\psi + c\bar{c}$ part with different charm quark mass m_c with the renormalization scale $\mu = 2m_c$ and $\sqrt{s} = 10.6$ GeV.

m_c(GeV)	$\alpha_s(\mu)$	$\sigma^{(0)}$(pb)	$a(\hat{s})$	$\sigma^{(1)}$(pb)	$\sigma^{(1)}/\sigma^{(0)}$
1.4	0.267	0.224	8.19	0.380	1.70
1.5	0.259	0.171	8.94	0.298	1.74
1.6	0.252	0.129	9.74	0.230	1.78

Table 5. Cross sections for $J/\psi + c\bar{c}$ part with different charm quark mass m_c. The renormalization scale $\mu = \mu^*$ is obtained by using BLM scale setting [43].

m_c(GeV)	$\alpha_s(\mu^*)$	$\sigma^{(0)}$(pb)	$b(\hat{s})$	$\sigma^{(1)}$(pb)	$\sigma^{(1)}/\sigma^{(0)}$	μ^*(GeV)
1.4	0.348	0.381	3.77	0.540	1.42	1.65
1.5	0.339	0.293	4.31	0.429	1.47	1.72
1.6	0.332	0.222	4.90	0.337	1.52	1.79
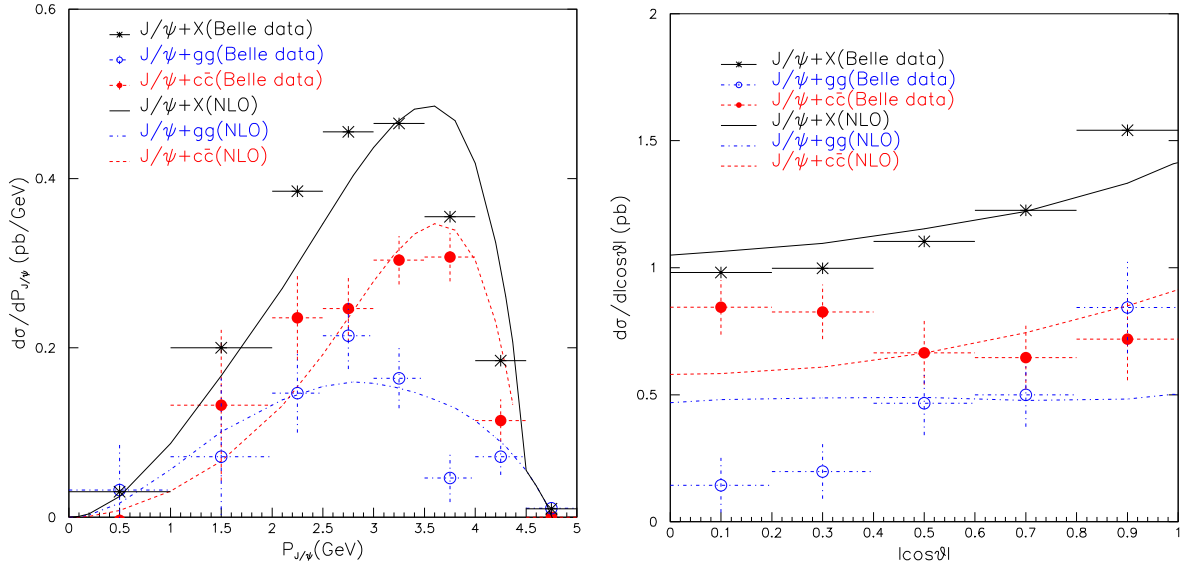
Figure 2. Momentum and angular distributions of inclusive J/ψ production at the B factories.

Corresponding total cross sections are shown in Table 5. From the relevant results listed, we can see that the K factors become smaller and the convergence for QCD perturbative expansions is improved.

Beside the total cross section, momentum and angular distributions of inclusive J/ψ production at the B factories are shown in Fig. 2, with $\mu = \mu^*$ and $m_c = 1.4$ GeV taken for the $J/\psi c\bar{c}$ part. It can be clearly seen that the momentum distributions roughly fits the experimental data, while for angular distributions, the predication for the total angular distribution agrees rather well with experimental measurement, but neither the $J/\psi + c\bar{c}$ nor the $J/\psi + X_{\text{non}-c\bar{c}}$ part can fit the experimental measurements.

2.3. Quarkonium production in hadron colliders

Based on NRQCD, the LO calculation predicts a sizable transverse polarization for J/ψ at high transverse momentum (p_t) region [44], while the CDF measurement [45] gives almost unpolarized result. Same problem was also found in bottomonium system. It is another challenging puzzle to our understanding on quarkonium production. Higher order corrections are highly expected to further clarify the situation. For J/ψ hadroproduction, up to LO in v^2, the intermediate state $(Q\bar{Q})_n$ can be $^3S_1^{[1]}, ^3S_1^{[8]}, ^1S_0^{[8]}$ and $^3P_J^{[8]}$. In Table 6, involving subprocess and number of corresponding Feynman diagrams are shown. The complexity and difficulty of corresponding calculation makes the work move slowly.

Table 6. Involving subprocesses and number of corresponding Feynman diagrams of quarkonium hadroproduction.

	$^3S_1^{[1]}$	$^3S_1^{[8]}$	$^1S_0^{[8]}$	$^3P_J^{[8]}$
$gg \to (Q\bar{Q})_n + g$	6/129	16/413	12/267	12/267
$gq \to (Q\bar{Q})_n + q$	—	5/111	2/49	2/49
$q\bar{q} \to (Q\bar{Q})_n + g$	—	5/111	2/49	2/49
$gg \to (Q\bar{Q})_n + gg$	60	123	98	98
$gg \to (Q\bar{Q})_n + q\bar{q}$	6	36	20	20
$gq \to (Q\bar{Q})_n + gq$	6	36	20	20
$q\bar{q} \to (Q\bar{Q})_n + gg$	6	36	20	20
$q\bar{q} \to (Q\bar{Q})_n + q\bar{q}$	—	14	4	4
$q\bar{q} \to (Q\bar{Q})_n + q'\bar{q}'$	—	7	2	2
$qq \to (Q\bar{Q})_n + qq$	—	14	4	4
$qq' \to (Q\bar{Q})_n + qq'$	—	7	2	2

2.3.1. CS channel In 2007, The NLO QCD corrections of CS channel are investigated by Campbell *et al.* [46]. The results show that the total cross section is boosted by a factor of about 2 and the p_t distribution is enhanced more and more as p_t becomes larger. Unfortunately, they do not have results for the polarization status. Soon after that, one-loop part of FDC package is completed. We perform a further study on this channel with FDC [47, 48]. Our predictions on the total and differential cross section are consistent with former work, and we also give predictions for the polarization for the first time. As shown in Fig. 3, the polarization is extremely changed from more transverse at LO into more longitudinal at NLO. Although it gives more longitudinal polarization than the recent experimental result [45] on the J/ψ polarization at Tevatron and the contribution of this channel is still too small to affect the total result, it sheds light on the solution to the large discrepancy.

2.3.2. S-wave CO channels Next step is on the S-wave CO channels [49, 50], which is thought to be dominant in total hadroproduction. Unfortunately, it is found that NLO QCD corrections in this channel are small. Neither the differential cross section nor the polarization is changed very much. Obvious gap is shown between theoretical predictions and experimental data. In the well established theoretical framework of NRQCD there still remains a narrow window which might make up the gap, one needs to investigate the NLO corrections to the P-wave CO channel $^3P_J^{[8]}$ and feed-down contribution from higher excited states.
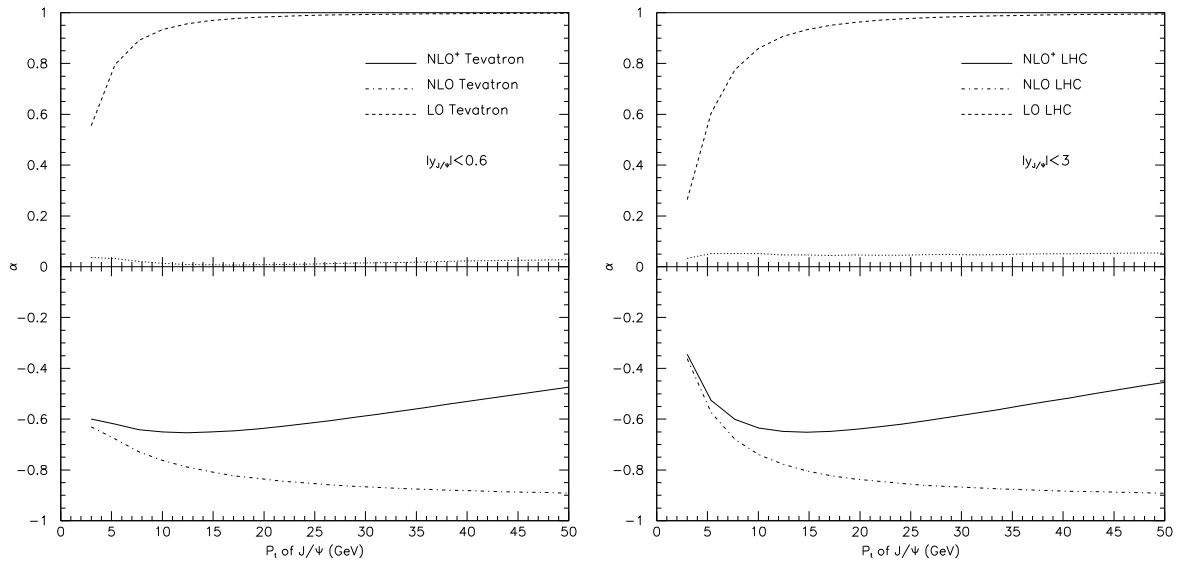
Figure 3. Transverse momentum distribution of polarization α at the Tevatron and LHC.

2.3.3. P-wave CO channel and complete NLO result But calculation on processes involving P-wave states is much more complicated than usual. It will cause abnormal denominator in the loop integral. FDC was unable to deal with such integral at that time. The study came to an impasse.

In 2010, complete NLO QCD corrections to J/ψ photoproduction are calculated by Butenschoen *et al.* [51]. This is for the first time such a complicated one-loop calculation involving P-wave state is completed. Later, NLO corrections for χ_{cJ} hadroproduction are studied by Ma *et al.* [52].

Then the complete NLO calculation for J/ψ hadroproduction is completed by Ma *et al.* [53, 54] and Butenschoen *et al.* [55] almost at the same time. Experiment measurement on the differential cross section of J/ψ hadroproduction can be described well, while NLO predictions for the polarization are still missing.

In 2011, a complete NLO result for polarization of J/ψ photoproduction is presented by Butenschoen *et al.* [56]. An overall agreement is found to recent measurement from H1 [57] and ZEUS [58]. The result inspires all the people concentrating on the J/ψ polarization puzzle.

In 2012, full NLO calculation on the polarization of J/ψ hadroproduction is completed by Butenschoen *et al.* [59]. Using the same LDMEs as those in above photoproduction case, their results disagree with CDF data [60, 45] at the Tevatron, but can account for ALICE data [61] at the LHC. Same work is also finished by Ma *et al.* [62] at the same time. Using a combined fit of J/ψ yield and polarization data from CDF, they find a possible solution to the polarization puzzle. The difference of the two groups comes from the fact that different NRQCD LDMEs are used in their works.

However, only direct J/ψ production is studied in both works, while there only exist polarization measurements for prompt (or even inclusive) J/ψ production [60, 45]. Moreover, it is known that among all the feeddown contributions to prompt J/ψ production, χ_{cJ} contributes more than $20 \sim 30\%$ [63, 64], and $\psi(2S)$ also contributes a small fraction, while others are negligible. The feeddown contribution is so large that it can drastically change the polarization results and must be considered.

Therefore, to test NRQCD factorization and solve (or clarify) the long-standing J/ψ polarization puzzle, it is a very important step to achieve the polarization predictions for prompt
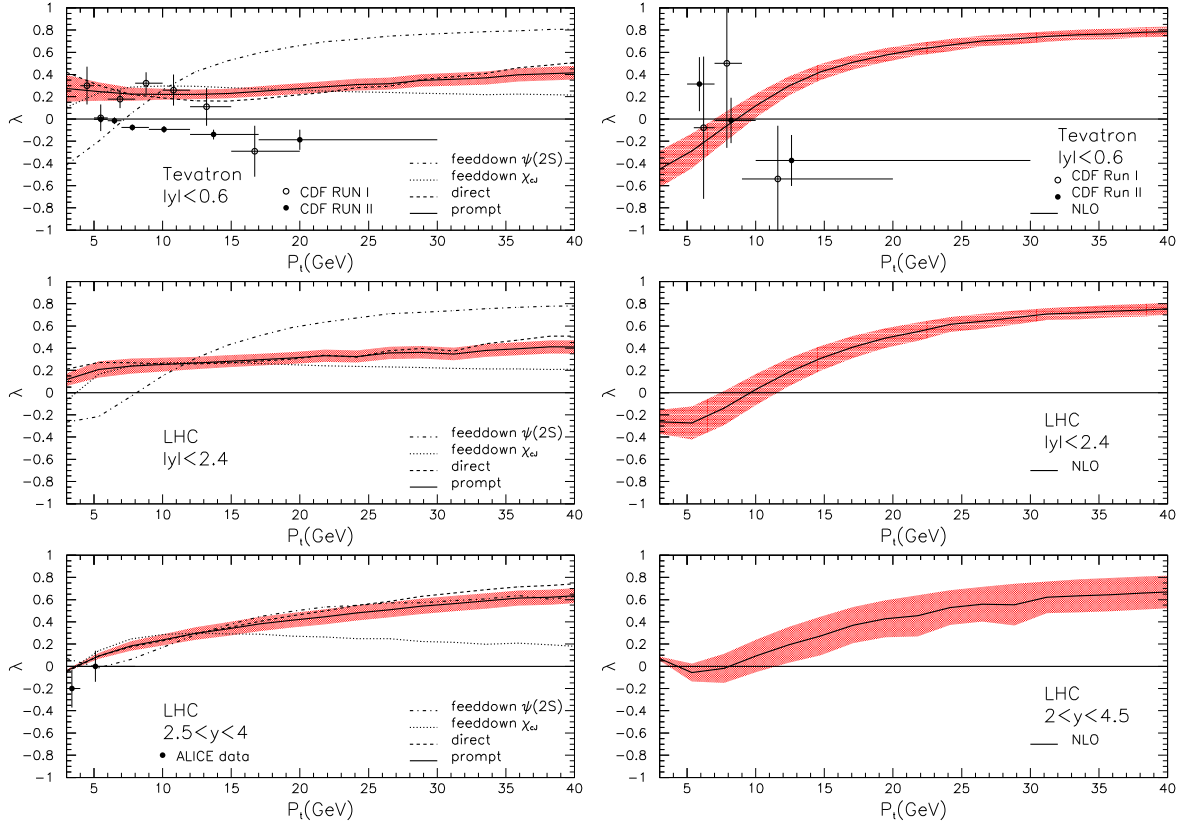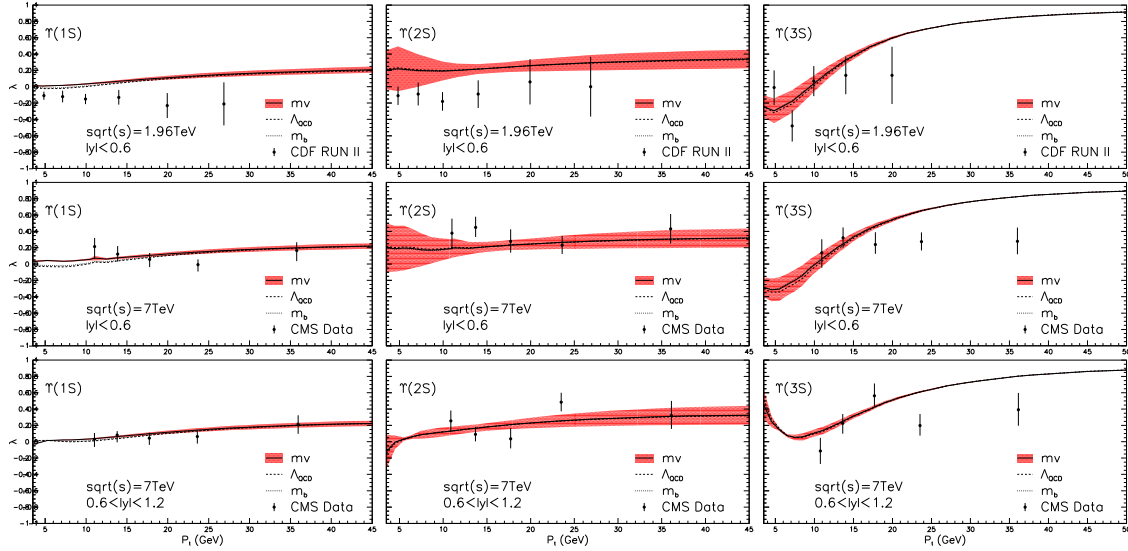
Figure 4. Polarization of J/ψ and $\psi(2S)$ hadroproduction at the Tevatron and LHC.

J/ψ hadroproduction. In 2011, FDC finished its upgrade on tensor reduction method. With this, we finished the study on polarization of prompt J/ψ hadroproduction [65]. In our result, not only all the direct channels, but also feed-down contribution from χ_{cJ} and $\psi(2S)$ are included. It is the first prediction for "prompt" J/ψ hadroproduction that can really compare with experimental data. As shown in Fig. 4, our predictions on the polarization of prompt J/ψ hadroproduction are in agreement with the CDF Run-I data, but in conflict with the CDF Run-II data. Up to now, it seems that in the framework of NRQCD, J/ψ polarization at the Tevatron and J/ψ cross section at the Tevatron/LHC can not be explained at same time.

For Υ hadroproduction, the CS channel [46, 47, 48] and S-wave CO channels have been investigated [50]. And NLO QCD correction to the yield for $\Upsilon(1S)$ via complete CO states (include ${}^3P_J^{[8]}$) is presented in Ref. [66]. The complete NLO QCD study on polarization of Υ hadroproduction has not yet been performed since there are more complicated feeddown and more difficulties to do experimental measurement than charmonium case at the Tevatron. However, the advantage for study on Υ is also obvious. Since bottom is almost three times as heavy as charm, both QCD coupling constant and v^2 are smaller in the bottomonium case. Then perturbative calculation on bottomonium should have better convergence in the double expansion of α_s and v^2 than that on charmonium. It is expected that the theoretical predictions on the polarization and yield of Υ at QCD NLO should be in better agreement with experimental measurement. Also it is found at the LHC that measurement on Υ is easier than that on J/ψ. As there are already polarization measurement on $\Upsilon(1S, 2S, 3S)$ by CMS [67], we perform a complete NLO study on the polarization and yield on Υ hadroproduction [68].

In Fig. 5, polarizations of $\Upsilon(1S, 2S, 3S)$ hadroproduction at the Tevatron and LHC are shown.

Figure 5. Polarization of $\Upsilon(1S, 2S, 3S)$ hadroproduction at the Tevatron and LHC.

For the polarization of $\Upsilon(3S)$, the theoretical prediction gives transverse polarization and goes far and far away from the data as p_t increases, from which we can conclude that the polarization can not be interpreted at LO in v^2 and NLO in α_s if unknown feeddown contribution from higher excited bottomonia is negligible. For $\Upsilon(1S, 2S)$, the predictions for polarization can well explain the CMS data, but still have some distance from the CDF data. The uncertainty from the badly known fraction of χ_{bJ} feeddown in the fits for $\Upsilon(1S, 2S)$ could be large which is not presented in the plots. Another uncertainty is from unknown feeddown contribution of higher excited states such as $\chi_{bJ}(3P)$. Therefore a further precise measurement on the fraction of χ_{bJ} feeddown or on direct Υ production will be very helpful to fix the polarization puzzle.

3. Summary
An automatic calculating system, FDC, is introduced. With its one-loop calculation part, and its upgrade on tensor reduction method, many important works in quarkonium physics have been studied. Most of them are very important in solve/clarify current puzzles in quarkonium physics.

4. Acknowledgments

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